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ON THE STANDARD SCALE OF TEMPERATURE IN THE INTERVAL 0° TO 100° C.

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I. OBJECT OF INVESTIGATION.

The standard scale of temperature in the interval $-35°$ to $+100°$, very generally adopted for scientific and technical work, is the scale of the constant volume hydrogen gas thermometer, defined in the following resolution of the International Committee on Weights and Measures, adopted October 15, 1887:

"The International Committee on Weights and Measures adopts as the standard thermometric scale for the international service of

weights and measures the centigrade scale of the hydrogen thermometer, having as fixed points the temperature of melting ice (0°) and of the vapor of distilled water boiling (100°) at standard atmospheric pressure, the hydrogen being taken at an initial manometric pressure of one meter of mercury—that is to say, $\frac{1000}{760} = 1.3158$ times the standard atmospheric pressure."

The scale adopted in this resolution refers to the scale defined by the hydrogen gas thermometer set up by Chappuis at the International Bureau of Weights and Measures, at Sèvres, near Paris, and compared by him with the scale[1] defined by four primary standard mercurial thermometers of French hard glass (verre dur), made by Tonnelot and bearing the Nos. 4428, 4429, 4430, and 4431. The relation between the mean scale defined by these four thermometers, which scale is known as the *mean verre dur scale*, and the Hydrogen Scale was thus established. From this known relation it is now therefore possible to express on the Hydrogen Scale, temperatures measured with a verre dur mercurial thermometer, provided the glass of which the thermometer is constructed is identical in its physical properties with the *verre dur* of the four standards that were directly compared with the hydrogen gas thermometer. This condition can not, however, be fully realized, as experience has shown that thermometers made up even out of the same ingot of glass differ somewhat in chemical composition and in physical properties, and therefore in the temperature scales which they define. While the magnitude of the variations in the scales, due to these causes, is quite small, it is yet definite and measureable, and for thermometers constructed of *verre dur* the deviation of the temperature scales from the *mean verre dur scale* may be as much as, or even exceed, ±0°01 in the interval 0° to 100°. To express temperatures on the International Hydrogen Scale with the highest attainable precision it is therefore necessary to apply to the corrected readings of a verre dur

[1] By the term "scale defined by a (mercurial) thermometer" is meant the temperatures found after correcting the observed readings for the variations in caliber of the tube, for the variation in the external pressure on the bulb from standard atmospheric pressure (760 mm), for the effect of the internal pressure on the bulb due to the column of mercury above the center of the bulb, for zero (which is the observed reading in melting ice, corrected for calibration, external and internal pressure, this corrected reading being applied, with changed sign, to the observed reading), and for the variation in the value of the fundamental interval from 100°.

thermometer further small supercorrections, which are the amounts by which the scale of the thermometer differs from the *mean verre dur scale* at the temperatures in question. These supercorrections must be determined either by direct comparison with the four primary standards of the International Bureau or by comparison with other primary standards whose relation to the *mean verre dur scale* has been so determined.

The Bureau of Standards was in possession of 16 primary standard mercurial thermometers, all constructed of *verre dur*, the calibration corrections, pressure coefficients, and fundamental intervals of which have been determined at the International Bureau at various times from 1885 to 1903. As the temperature scales defined by these thermometers would very likely differ slightly from one another, one of the objects of the present investigation was to determine the magnitude of these differences; in other words, to determine the *supercorrections* that must be applied at different temperatures to the scale defined by each thermometer to reduce to temperatures on the mean scale of all the thermometers, and thus to establish a standard scale of temperature for future use and reference with as high a degree of precision as possible.

The advantages resulting from the use of the same standard scale of temperature are of course at once obvious. Only a few years ago, before the introduction of the International Hydrogen Scale, investigators frequently spent far more time in setting up a gas thermometer and establishing a temperature scale to which to refer their measurements than in carrying out their experiments. A familiar illustration is furnished by the classical determinations by Rowland of the mechanical equivalent of heat and the capacity for heat of water. A further great advantage in a single standard scale at once available to every investigator, aside from the great saving in time, is that all measurements expressed on that scale, made by different methods and by different observers, can be compared with one another with the greatest advantage, and outstanding differences very often explained. Thus a comparison of the best determinations of the mechanical equivalent of heat, carried out by mechanical and electrical methods, showed a difference amounting to about 1 part in 400, and the two methods gave a different variation of the capacity for heat of water with the temperature. When,

however, the temperature scale used by Rowland, based on his air thermometer, was reduced to the International Hydrogen Scale, which was used by Griffiths and by Schuster and Gannon in their experiments, the curves expressing the relation between the capacity for heat of water and the temperature, found by the two methods, were parallel and showed a constant difference of about 1 part in 400 in the interval 15° to 25°. An analysis of the factors entering into the determinations at once pointed to a possible error in the electrical units in terms of which the energy measurements were expressed, which was almost simultaneously confirmed by the redeterminations of the electromotive force of the standard Clark cell. When the electrical determinations were then corrected by using the newly found value of the electromotive force of the Clark cell, the results found by the two methods were brought into agreement to within about 1 part in 2000.

It is therefore highly desirable that the standard scale of temperature to be used by this Bureau should be as nearly as possible identical with the International Hydrogen Scale. By intercomparisons of the standard thermometers at a number of temperatures, and thus determining their supercorrections to the mean scale of all of them, it was possible to fix a very definite temperature scale. As several of these thermometers have been directly compared with the standards of the International Bureau it was therefore possible to determine any outstanding difference between the mean scale defined by these thermometers and the mean verre dur scale defined by the standards of the International Bureau, which scale serves to fix and to reproduce the International Hydrogen Scale.

The results of the redetermination of the constants of these thermometers and their intercomparisons in various groups at every 10° in the interval 0° to 100°, may be briefly summed up in the statement that the mean scale defined by the 16 primary standard thermometers included in the investigation is in agreement with the mean verre dur scale of the International Bureau and therefore serves to reproduce the International Hydrogen Scale to within about 0°.002, which is probably within the limits of accuracy at present attainable in mercurial thermometry.

II. MERCURIAL THERMOMETRY.

The instrument most generally used for temperature measurement within the interval 0° to 100° (or even 500°) is the mercury-in-glass thermometer, the two fixed points of which, 0° and 100° on the centigrade scale, are defined respectively as the temperature of melting pure ice and the temperature of the vapor of boiling pure water, both at standard atmospheric pressure.[2] The scale is then fixed by dividing the volume included between these two fixed points into 100 equal parts.

The scale of temperature thus defined is found to vary (*a*) with the composition of the glass, such slight changes in composition as are brought about in working the bulbs in the blast flame being often sufficient to produce a noticeable effect on the temperature scale defined by the thermometers, and (*b*) even with glass of the same composition in different physical conditions, as to strains. The mercury-in-glass scale is therefore different for every different kind of glass used. Thermometers made up at different times, of glass of the same trade name, are found to differ by as much as 0°.02 in the temperature scales which they define.[3]

The only scale of temperature independent of the physical properties of a particular substance is the absolute thermodynamic scale introduced into physical science by Lord Kelvin. The scale defined by a perfect gas would be in agreement with this scale. The scale defined by the more permanent gases is a very close approximation to the absolute scale. For this reason, as well as on account of the large coefficient of expansion of gases and the large range throughout which they can be used, the scale of the gas thermometer is adopted as the standard of reference in all temperature measurements. Our knowledge of the slight departure of this scale from the absolute thermodynamic scale is based on experiments showing the departure of gases from the laws of a perfect gas. The first experimental realization of the absolute scale is due to Thomson and Joule[4] in

[2] I. e., a pressure of 760 mm of mercury at 0° C, latitude 45°, sea level.

[3] Grützmacher: Wiss. Abhandl. der Phys.-Tech. Reichsanstalt, **3**, p. 256; 1900.

[4] Thomson and Joule: On the Thermal Effect of Fluids in Motion, Phil. Trans. Roy. Soc. **143**, p. 357; 1853: **144**, p. 321; 1854: **152**, p. 579; 1862.

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their porous plug experiments on the cooling (or heating) of a gas accompanying its free expansion.

The departure of the scales of mercury-in-glass thermometers, made of the different kinds of glasses that are now very generally used (verre dur, Jena 16^{III}, Jena 59^{III}, and English-Kew glasses), from the scale of the standard gas thermometer has been determined by a number of investigators,[5] so that it is now possible to express on the gas scale, temperatures measured with mercurial thermometers made of these glasses. A thermometer so constructed that it is capable of defining within itself a scale of temperature is called a *primary standard.* Experience has shown that the length of one degree which best satisfies all the requirements for a primary standard in the interval 0° to 100° is about 6 or 7 mm (not less than 5). This length of degree divided into 0°.1 intervals, when used with a small telescope magnifying 6 to 10 times, permits of eye estimates to 0.01 or 0.02 of such an interval (i. e. 0°.001 or 0°.002). An increase in the length of a degree and a finer subdivision would increase the accuracy of reading, which, however, would be more than offset by other sources of error; for to increase the length of a degree a larger bulb would be necessary, which would increase the lag of the thermometer, unless a thinner wall bulb or a finer capillary were used. In both of the latter cases the effect of the variable capillary pressure of the mercury meniscus would give rise to greatly increased uncertainties in the indications of the thermometer. For

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[5] For the French hard glasses: Chappuis: Trav. et Mem. du Bur. Int. des Poids et Mesures. **6**; 1888.

For the Jena 16^{III} normal, 59^{III} borosilicate, and 122^{III} baryt-borosilicate glasses; Wiebe and Böttcher: Zs. für Instrumentenkunde, **10**, p. 245; 1890. Grützmacher: Wied. Ann., **68**, p. 769; 1899. Thiesen, Scheel, und Sell: Wiss. Abhandl. der Phys.-Tech. Reichsanstalt, **2**, pp. 1–71; 1895; Zs. für Instrumentenkunde, **15**, p. 433; 1895. Mahlke: Wied. Ann., **53**, p. 965; 1894. Scheel: Wied. Ann., **58**, p. 168; 1896. Lemke: Zs. für Instrumentenkunde, **19**, p. 33; 1899. Grützmacher: Zs. für Instrumentenkunde, **15**, p. 250; 1895; also Greiner and Friederich's "Resistenzglas."

For the older Thüringen glasses and English-Kew glass: Grunmach: Metron. Beiträge der Kais. Normal-Aich. Kommission, Berlin, No. **3**, p. 54; 1881. Marek: Zs. für Instrumentenkunde, **10**, p. 283; 1890. Wiebe: Zs. für Instrumentenkunde, **10**, p. 438; 1890. Grützmacher: Wiss. Abhandl. der Phys.-Tech. Reichsanstalt, **3**, p. 231. Balfour Stewart: Phil. Trans. Roy. Soc., **153**, p. 425; 1863. Chree: Phil. Mag., **45**, p. 225; 1898. Guillaume: Determination du Rapport du Yard au Mètre, Bur. Int. des Poids et Mes.; 1896. Schloesser: Zs. für Instrumentenkunde, **21**, p. 296; 1901. Harker: Proc. Roy. Soc., A, **78**, p. 225; 1906.

higher temperatures the length of a degree should be less. In order not to unduly increase the length of the thermometer the scale need only include the region in which it is intended for use, e. g., 0° to 50°, 100° to 200°, etc. It must, however, be so constructed with suitable auxiliary reservoirs in the stem that it contains the two fixed points 0° and 100°, and further, that the volume of any part of the stem can be referred to the fundamental volume between the 0° and 100° marks.

In the practical construction of thermometers it is not possible to realize the ideal conditions, viz, exact location of the 0° and 100° marks on the stem, and subdivision of the included volume into 100 equal parts. In the construction of laboratory thermometers this is approximately done by calibrating the tube by moving mercury threads along the capillary and observing their lengths in various positions and thus fitting the graduation to allow for irregularities of the bore. By this method good laboratory thermometers can be constructed accurate to a few hundredths of a degree. For primary standards, however, this method of construction is not permissible, for here the corrections for variation in caliber must be known to a far higher degree of accuracy and must be determined by the most careful *calibration of the tube.*[6] The amount of work involved in this calibration is greatly increased if the graduations are irregularly spaced. The tube of which the thermometer is made must be carefully selected by a preliminary calibration and must be very uniform in cross section. The greatest difference of the calibration corrections should not exceed 0°.2 or 0°.3 at most. After the positions of the fixed points are determined the space between must be divided

[6] Full details of the methods of calibration of mercurial thermometers will be found in the following references: Marek: Carl's Rep., **15**, p. 300; 1879; adaptation of method of Hansen, Abhandl. d. math.-phys. Classe der K. Sächs. Gesellschaft der Wiss.; 1874. Thiesen: Carl's Rep., **15**, p. 285, p. 678; 1879. Report of Committee of the British Association for the Advancement of Science, B. A. Report, pp. 145–204; 1882. Benoit: Trav. et Mem. du Bur. Int. des Poids et Mes., **2**, p. C35; 1883. Broch: Ibid, **5**, p. 3; 1886. This and the preceding reference relate to the calibration of scales. Guillaume: Ibid, **5**, Etudes Thermometriques, p. 5; 1886; Thermometrie de Precision, p. 40, Gauthier-Villars et Fils, 1889. Pernet, Jaeger und Gumlich: Wiss. Abhandl. der Phys.-Tech. Reichsanstalt, **1**; 1894. Pernet: Viertel Jahrschrift der Naturforsch. Gesellschaft, Zurich, **41**, p. 128; Anm. 1 Grützmacher: Wiss. Abhandl. der Phys.-Tech. Reichsanstalt, **3**, p. 245; 1900.

into parts of equal length. For this purpose a dividing engine should be used whose screw has been very carefully studied for progressive and periodic errors. As the length of a degree is generally 5 to 7 mm the accidental errors of ruling should be kept within a few thousandths of a millimeter. The graduation marks on the stem should be very fine and clear, and their thickness should not be greater than 0.05 of the smallest scale interval and should preferably be less.

As the indications of a thermometer are influenced by the pressure on the outside of the bulb (atmospheric pressure and the pressure of the medium in which the bulb is immersed) and by the pressure from within due to the mercury column and capillary forces, it is necessary to determine the *external* and *internal pressure coefficients* so that the indications of the thermometer can be reduced to standard conditions, i. e., to an external pressure of 760 mm and an internal pressure of zero (really the somewhat variable pressure of the mercury meniscus) corresponding to the horizontal position of the thermometer.

As it is not possible for the maker to locate without error the fixed points, the value of the interval included between these two points must be determined with the greatest care by observing the reading of the thermometer in steam and the corresponding barometric pressure; and immediately after, before any appreciable recovery of the zero shall have taken place, the reading in melting ice. This gives the necessary data for determining the *fundamental interval* of the thermometer, i. e., the number of scale degrees[7] of the thermometer between the temperatures defined by the two fixed points, 0° and 100°. The error in the fundamental interval should not exceed 0°.1.

The stem of the thermometer should be transparent, so that errors of parallax can be avoided by taking the mean of readings with scale before and with scale behind the mercury column, the line of sight remaining unchanged; for this reason the use of an enamel-back stem is not desirable in a primary standard thermometer. In thermometers of the inclosed scale type errors of parallax are

[7] I. e., corrected for variation in caliber and for internal and external pressure.

avoided by adjusting the line of sight so that the portion of the graduation seen through the fine capillary stem is continuous with the graduation seen on either side of it.

To better understand the methods adopted in the intercomparisons described in this paper it will perhaps be worth while to briefly consider the thermal properties of glasses and especially the effects of thermal hysteresis on the indications of thermometers.

Glasses exhibit in varying degree the phenomena of *thermal hysteresis.* When a thermometer is heated to a definite temperature it expands to its final equilibrium condition in a few minutes for the better thermometric glasses and in an hour or so for the inferior glasses. When the thermometer is again cooled to its original temperature it takes from many days to many months for the glass to again contract to the equilibrium condition corresponding to that temperature. These phenomena manifest themselves in the so-called *depression of the zero*[8] and *recovery of the zero.*

The glasses[9] used before the introduction of the verre dur, Jena 16^{III} normal, and Jena 59^{III} borosilicate glasses, nearly all showed large zero depression, ranging from a few tenths of a degree to a degree or more. The above glasses have depressions of about 0°.10, 0°.08, and 0°.03, respectively. The Jena 59^{III} borosilicate glass is the best thermometric glass yet brought out. In addition to the small zero depression after heating, the scale of temperature which it defines nowhere differs from the scale of the hydrogen thermometer in the interval 0° to 100° by more than 0°.03. This glass can also be used for temperatures as high as 500°. The investigations of R. Weber[10] and of Wiebe[11] have shown that glasses in which are present the oxides of both sodium and potassium show relatively

[8] As a measure of the zero depression is taken the difference in the ice point corresponding to long continued (some weeks) exposure at 0°, and the ice point taken immediately after the thermometer has been at 100°, before any appreciable recovery of the zero shall have taken place.

[9] For a complete discussion of the thermometric behavior of some of the older glasses reference is made to Grützmacher: Wiss. Abhandl. der Phys.-Tech. Reichsanstalt, **3**, p. 231; 1900.

[10] R. Weber: Sitzungsberichte d. K. Preuss. Akad. d. Wiss., Dec. 13; 1883.

[11] Wiebe: Ibid., July 17, 1884; Nov. 12, 1885. Zs. für Instrumentenkunde, **6**, p. 167; 1886.

large depressions in comparison with glasses in which only one of these oxides is found, and especially so if the two are present in about equal proportions. The same statement holds for the elastic hysteresis exhibited by the glasses after undergoing mechanical deformation of any kind.[12]

On account of these phenomena of thermal hysteresis, the observed reading of a thermometer depends not alone on its temperature, but also on its previous thermal history. The effect of the consequent variations of zero on the observed readings is eliminated by the procedure due to Pernet,[13] which consists in using as the zero from which the temperature is to be reckoned the reading found in melting ice, immediately after the temperature measurement in question, the fundamental interval being determined in the same way by using the ice point observed immediately after the steam-point observations.

Other changes which affect a mercury-in-glass thermometer may be broadly grouped under the head of *secular* and *annealing changes.*

There is a slow contraction of the glass and consequent rise of the ice point which goes on for years. With thermometers of suitable glass that have been properly annealed this change is very small, and should not exceed 0°.1 in some years for thermometers that are used at ordinary temperatures. The larger part of this change takes place in the first few months after the thermometer is made up.

A discussion of the annealing changes and experimental data relating thereto will be found in a paper by one of the authors,[14] so that it will only be necessary to briefly refer to this subject here. The strains set up in the glass in making the thermometer, which ordinarily only slowly disappear, are relieved by a suitable annealing at high temperatures, resulting in a contraction of the glass and consequent rise of the ice point. Unless the thermometer has been properly annealed a small rise of the ice point will be observed after each heating. Hence in experiments on the zero depression

[12] G. Weidmann: Dissert., Jena; 1886. Wied. Ann., **24**, p. 214; 1886.

[13] Pernet: Trav. et Mem. du Bur. Int. des Poids et Mesures, **1**, p. B.12; 1881.

[14] Dickinson: Bureau of Standards Bulletin, **2**, p. 189; 1906.

it is necessary to work with well-annealed glass, or the phenomena will be partly masked by the rise of the ice point. Schott[15] has investigated the effect of annealing and has found that it acts by relieving the strains, as was shown by the almost complete disappearance of double refraction in glass after annealing. Thermometers intended for use at high temperatures should be annealed at a temperature of about 450° for a period of 200 hours, which treatment should be followed by a period of slow cooling extending over a day or two. The total rise of the ice point for this process of annealing will be about 25° or 30°. The contraction of the glass, which results from the annealing of a thermometer, takes place in the stem as well as in the bulb, and produces an increase in the fundamental interval. As the volume included between the two fixed points is approximately one-sixtieth of the volume of the bulb, it follows that, if the stem undergoes the same proportional change in volume as the bulb, the increase in the fundamental interval should be about one-sixtieth of the observed rise in the ice point. Experiment shows that the change in the fundamental interval is more nearly one-thirtieth of the observed rise of the ice point. This is very probably due in part to the fact that the strains present in the thick glass of which the stem is made are greater than in the thin wall tubing of which the bulb is made, so that the change in volume, on annealing, is relatively greater. However, an increase in the fundamental interval is observed when only the bulb is annealed. The coefficient of expansion of glass is greater in the strained than in the unstrained condition.[16] When the strains which are present in the glass are relieved by the annealing the coefficient of expansion is diminished, which results in an increase in the fundamental interval of the thermometer. When, however, a thermometer has been well annealed the subsequent change in the fundamental interval is very small, even when used at high temperatures, and when used in the interval 0° to 100° does not much

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[15] Schott: Zs. für Instrumentenkunde, **11**, p. 330; 1891.

[16] Schott: Vortrag im Verein zur Beförd. d. Gewerbfleisses, Berlin; 1892. Hovestadt: Jenaer Glas, p. 235, published by Gustav Fischer, Jena, 1900; this book contains very complete information on the physical and chemical properties of the various Jena and some other glasses.

exceed the limits of experimental error within which this constant can be determined.[17]

For primary standards used within this interval the relative changes in the caliber of the tube are almost within the limits of precision to which the calibration corrections can be determined.[18] Long continued use apparently results in a very small decrease in the volume of the small auxiliary reservoir relatively to the remainder of the capillary tube, as is to be expected from the strained condition of the glass in the region of this reservoir.

III. CONSTANTS AND STANDARDIZATION OF THERMOMETERS.

The primary standard thermometers available for these intercomparisons were:

Tonnelot Nos.	4331	4336	4332	4623	4334	4624	4335
Baudin Nos.	15282	15962	15555	15963	15583		
	16016		16017		16018		
Tonnelot No.	11801						

Full details of the types, ranges, and dimensions of the thermometers are given in Fig. 8 and in Table XXI in the Appendix to this paper.

These thermometers are all made of French verre dur, a lead-free glass containing about 11 per cent of the oxide of sodium (Na_2O), and only a small trace (0.3 or 0.4 per cent) of the oxide of potassium (K_2O).[19]

The series of Tonnelot thermometers Nos. 4331–4336 were made by Tonnelot, of Paris, in 1884. Of this series, 4333 was broken before the beginning of this investigation. Nos. 4623 and 4624 were made in 1888. This set of eight thermometers accompanied the Interna-

[17] Guillaume: Trav. et Mém. du Bur. Int. des Poids et Mesures. **5**; 1886.

[18] Guillaume: loc. cit., pp. 63–67; and Grützmacher: Wiss. Abhandl. der Phys.-Tech. Reichsanstalt, **3**, p. 242; 1900.

[19] Tornöe: Trav. et Mém. du Bur. Int. des Poids et Mesures, **5**; 1886.

tional Prototype Meters Nos. 21 and 27, and the International Prototype Kilograms Nos. 4 and 20, which were awarded by lot to the Government of the United States. These thermometers are graduated into 0°.1 and include on their scales the interval 0° to 50°, above which are an auxiliary reservoir and a small region of the scale including the 100° point. The length of 1° varies for the different thermometers from 6.56 mm to 7.02 mm. The thickness of the graduation marks, which are sharp and clear, varies from about 30 to 45 microns (0.04 to 0.06 of an interval), being less in the more recently constructed thermometers.

Thermometers Nos. 15282, 15555, 15583, 15962, 15963, 16016, 16017, and 16018 were made by Baudin, of Paris, at various dates from April, 1900, to November, 1903. All of these thermometers are graduated into 0°.1, and, as regards fineness of graduations and perfection of workmanship, leave nothing to be desired. Nos. 15282, 15962, and 15963 include the range 0° to 100°, the length of 1° being 5.913, 5.914, and 5.878 mm, respectively. Nos. 15555 and 15583 include the range 0° to 50°, the length of 1° being 8.252 and 7.954 mm. Nos. 16016, 16017, and 16018 include the range 50° to 100°, the length of 1° being 7.260, 7.236, and 7.196 mm, respectively. The thickness of the graduation marks is about 18 microns (0.02 to 0.03 of an interval).

Tonnelot No. 11801, which was kindly loaned for this investigation by Professor J. S. Ames, of the Johns Hopkins University, was made by Tonnelot in 1895, and includes the range 0° to 100°. The length of 1° is 5.858 mm.

All of these thermometers have been submitted to an exhaustive study for errors of graduation, calibration corrections, external and internal pressure coefficients, and fundamental interval corrections at the International Bureau of Weights and Measures under the supervision of Dr. Ch. Ed. Guillaume. Tonnelot No. 11801 and Baudin Nos. 15962, 15963, 16016, 16017, and 16018 have in addition been compared with the primary standard mercurial thermometers of that Bureau in order to determine the small deviation of the verre dur scale which they define from the mean verre dur scale of the International Bureau, which scale fixes the International Hydrogen Scale of Temperature.

Tonnelot No. 11801 has also been compared by Dr. W. S. Day[20] and by Mr. F. Mallory and one of the authors[21] with the thermometers used by Rowland in his determinations of the mechanical equivalent of heat, as well as with a Callendar-Griffiths platinum resistance thermometer, so that the scale defined by Tonnelot 11801 is directly connected with the gas scales defined by the air thermometers of Rowland and of Callendar and Griffiths, as well as with the International Hydrogen Scale.

Calibration corrections.—The details of the methods employed in the calibration of the thermometers at the International Bureau, together with the tables of the calibration corrections, are given in the Appendix, pp. 721–728.

We have not deemed it necessary to repeat in detail the calibration of these thermometers, as any small errors in the calibration corrections would be taken care of in the supercorrections found from the intercomparisons. As the calibrations of some of the Tonnelot thermometers were made nearly twenty years ago, it seemed possible that measurable changes might have taken place in the relative volumes of different portions of the stem. If such were the case, one would expect to find the greatest changes in the calibration correction at 50° for those thermometers having a 50° auxiliary reservoir, as the strains called into existence in the construction of the thermometers in the region of this enlargement are probably greater than, or at least different from, the strains present in the remainder of the stem, so that the disappearance of these strains might be expected to affect the volumes of the tube and the reservoir differently. Furthermore, on account of the difference in area of the two 50° intervals, any surface changes on the interior walls would be most pronounced for such a thermometer. Some check determinations of the calibration corrections at 50° for thermometers of this type gave results that differed from the values given in the certificates by amounts which were within the limits of experimental error (0°.002).

[20] W. S. Day: Phys. Rev., **6**, p. 193; 1898. Phil. Mag., **46**, p. 1; 1898.

[21] Waidner and Mallory: Phys. Rev., **8**, p. 193; 1899. Phil. Mag., **48**, p. 1; 1899.

Pressure coefficients.—The values of the pressure coefficients of the thermometers, as given in the certificates, are summarized in Table I, where

β_e = external pressure coefficient = change of reading in scale degrees produced by a change in pressure of 1 mm of mercury.

β_i = internal pressure coefficient = change of reading in scale degrees produced by a change in pressure of 1 mm of mercury.

The method employed in these determinations at the International Bureau is to inclose the thermometers in a glass tube, containing sufficient mercury to cover the bulb of the thermometer, the remainder of the tube being filled with glycerine. Observations are then made with slowly rising temperature, as the pressure within the tube is varied by alternately connecting it with an exhausted receiver and opening it to the atmosphere. This gives the necessary data for the determination of the external pressure coefficient, β_e. The internal pressure coefficient is obtained from the relation

$$\beta_i - \beta_e = 0.000\,015\,4.$$

In our own determinations of the fundamental intervals, described later, the thermometers were read in steam in both horizontal and vertical positions, from which and the known dimensions of the thermometers the internal pressure coefficients were directly obtained. The values of β_e obtained in this way,[22] by use of the above relation, are included in Table I.

[22] In all of the computations of the values of β_e from the observations in steam, the necessary corrections were made for the variation in the calibration corrections and for the difference of level of the bulb in the horizontal and vertical positions of the thermometer in the steam-point apparatus.

TABLE I.

Pressure coefficients.

Thermometer	From Certificates of the Bureau International	From the observations in steam
Tonnelot No. 4331	0.0001145	0.0001163
" " 4332	.0001080	.0001058
" " 4334	.0001192	.0001214
" " 4335	.0001202	.0001167
" " 4336	.0001219	*.0001215
" " 4623	.0001282	.0001255
" " 4624	.0001228	.0001202
" " 11801	.0001159	.0001147
Baudin " 15282	.0001536	.0001525
" " 15555	.0001483	.0001463
" " 15583	.0001336	.0001316
" " 15962	.0001238	*.0001248
" " 15963	.0001345	(†)
" " 16016	.0001199	*.0001216
" " 16017	.0001266	.0001265
" " 16018	.0001217	.0001240

*The external pressure coefficient, β_e, was determined directly for 4336, 15962, and 16016, in an apparatus similar, in all essential particulars, to that used at the Bureau International. The bulb of the thermometer was immersed in mercury, the remainder of the tube being filled with water. The two independent determinations by Mr. E. F. Mueller and one of the authors have given the following values:

Observer.	No. 4336	No. 15962	No. 16016
H. C. D.	0.0001211	0.0001244	0.0001216
E. F. M.	.0001219	.0001253	.0001216
Mean	0.0001215	0.0001248	0.0001216

The values found from the observations in steam, for these three thermometers, were not entirely satisfactory, and differed by several per cent from the above values, being lower for 4336 and higher for 15962 and 16016.

† No. 15963 was broken during the intercomparisons at 80°.

The agreement between the values of β_e, as given in the certificates and as found from the observations in steam, is very satisfactory. Our values of the coefficients are, on the average, smaller by 0.0000006 (about 0.5 per cent), which corresponds to less than 0°.0004 in the correction for internal pressure at 100°. In this con-

nection it is interesting to note that Thiesen, Scheel, and Sell[23] have shown that the heating and cooling of the liquid in which the thermometer is immersed, produced by the variations in pressure, will in general give high values for the pressure coefficient. This effect is greater for glycerine than for mercury or water, and vanishes for water at 4° C. These investigators found that when the determinations are made with the bulb immersed in mercury the value of the pressure coefficient is increased, due to the above cause, by about 0.0000014 (i. e., about 0.8 per cent), and recommend that the experiments on the determination of external pressure coefficient be carried out in a tube filled with water, in which case the correction due to the above cause is negligible.

In the observations in steam the readings are taken first in the horizontal and then in the vertical position (see below), so that the reading in vertical position is obtained under the disadvantageous condition of falling meniscus. For this reason, as well as on account of the great number of readings that can be obtained in the direct determination of β_e in a short time, the determinations of this constant with the well-known Marek pressure apparatus, as used at the International Bureau, is entitled to greater weight than the values given by the observations in steam. We have therefore used the values of the pressure coefficients given in the certificates, and which are most satisfactorily corroborated by our own determinations, the maximum difference only in one case amounting to 0°.0015 in the value of the internal pressure correction at 100°.

Determination of fundamental intervals.—The fundamental intervals of the thermometers were determined by observations in steam, with divisions before and divisions back, in horizontal and vertical positions, followed by similar observations in ice in vertical position.

Observations in steam.—The observations in steam were made in the International Bureau type of boiling-point apparatus devised by Chappuis, and shown in Fig. 1, which consists of a double wall steam jacket, the steam being led from the boiler, through the axis on which the apparatus turns, up the central tube in which the thermometer is found and down the outer annular space into the condenser, whence the condensed steam is returned to the boiler. The

[23] Thiesen, Scheel, and Sell: Wiss. Abhandl. der Phys.-Tech. Reichsanstalt, 2, p. 7; 1895.

excess of pressure within the apparatus was measured by the small water manometer. By a careful preliminary study with several such manometers, inserted in the thermometer space from above as well as in the axis, with the inner opening of the manometers in different directions, it was made certain that the slight excess of pressure within the apparatus as measured by the manometer (which rarely attained 3 mm of water) was a true measure of the excess of pressure in the region in which the bulb was found, and that the readings were not appreciably affected by the dynamic action of the flowing steam. The action of the manometer was also tested by mounting within the apparatus a sensitive platinum resistance thermometer, and varying the excess of pressure from a fraction of a millimeter to 10 mm of water. The resistance thermometer at once responded to every change in pressure by the calculated amount, to within 0°.001, thus further indicating that there was no appreciable superheating of the steam even for quite rapid steaming.

The top of the hypsometer was closed with a thin rubber disc, through a hole in which the thermometer was inserted. It was thus possible to expose nearly all the mercury thread to the steam except a few tenths of a degree, in and above the rubber, to permit of reading. With this arrangement, however, the mercury distills from the end of the column into the cooler region above it, so that the reading changes with time. The error from this source is in part eliminated, as the reading in ice, which is taken immediately after the observations in steam, will also be lowered. To avoid this source of error Pernet, Jaeger, and Gumlich[24] mounted above the hypsometer a small glass bell jar so that the thermometer was entirely inclosed in steam. To avoid the difficulties of reading the thermometer through the wet glass, we have sought to attain the same ends by adding to the apparatus a simple device which may be termed the *emergent stem heater* shown diagrammatically in Fig. 2 and in position on the hypsometer in Fig. 1. It consists of a brass tube, A, closed at the top with a removable cover, B, through which the rod R passes, and from which the thermometer is suspended. A current of air from the laboratory pressure system is passed

[24] Pernet, Jaeger, and Gumlich: Wiss. Abhandl. der Phys.-Tech. Reichsanstalt, 1; 1894.

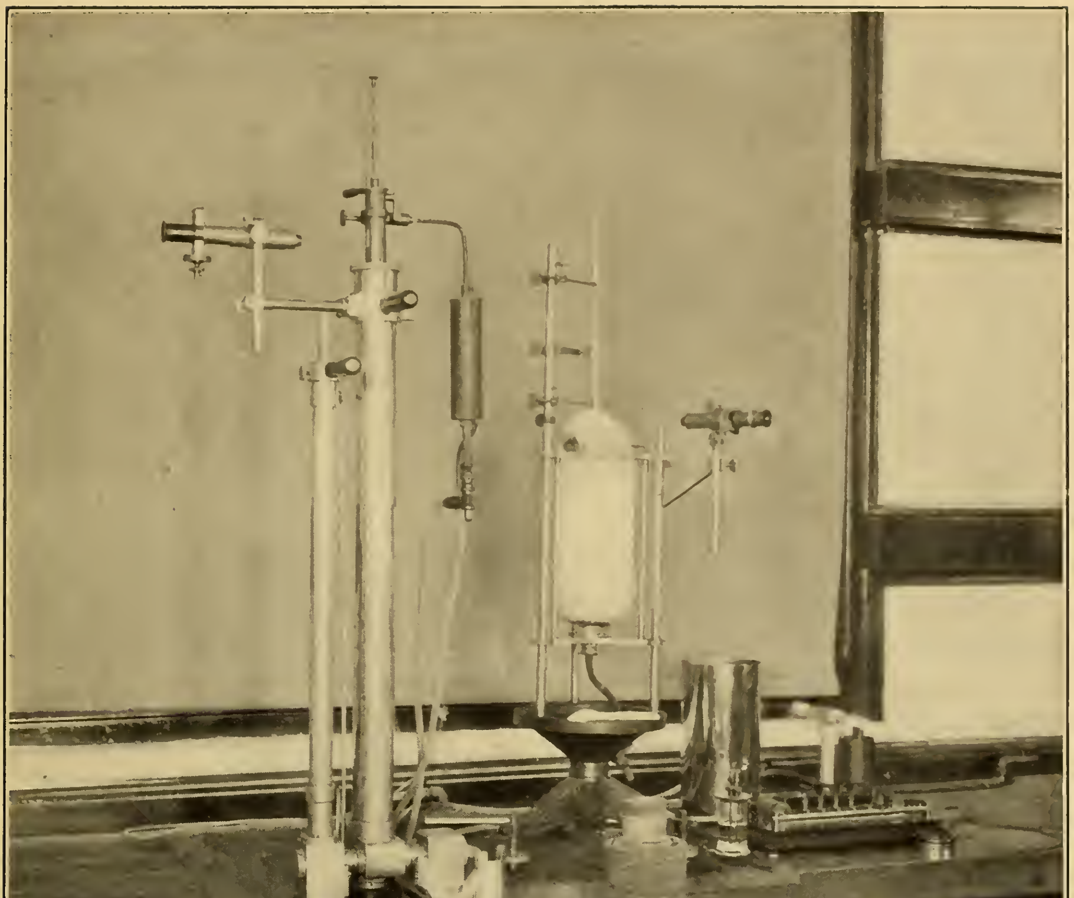

Fig. 1.—*Steam-Point and Ice-Point Apparatus.*

through the coiled copper pipe C and heated to the temperature of 100° to 105° by means of the Bunsen burner B*b*, and is then discharged into the space A, inclosing the emergent stem of the thermometer, T, the temperature of the air being indicated by a small thermometer, *t*, and being controlled by valves in the air and gas supply pipes. When once adjusted, with only occasional attention the temperature of the air could be kept at practically 100° for any length of time, and all sources of error incident to evaporation were avoided. The standard thermometer, T, was read by means of a telescope through the front plane glass window, W_1, and the scale of the thermometer was illuminated by light reflected through the rear window, W_2, by means of a piece of milk glass.

Pure water was used in the steam-point apparatus; a number of small quartz crystals in the bottom of the boiler served as nuclei to facilitate smooth boiling. Previous to being placed in position in the steam bath, the thermometers were exposed to a temperature of 100° in a Regnault hypsometer of the usual laboratory form for a period of 10 to 15 minutes.

The observations were taken first with the thermometer in horizontal and then in vertical position. This sequence of observations is necessary in the usual method of determining the steam point, where distillation is present, in order to avoid the possible error due to the distilled mercury which would be reunited with the column if the thermometer were turned from vertical to horizontal position. This procedure has one disadvantage, that the reading in vertical position is made with falling meniscus. This source of error may, however, be avoided by withdrawing some 10° or 15° of the stem from the steam bath for an instant and then pushing the thermometer back into position. When the emergent stem heater was used and distillation was avoided, it would therefore have been better to have taken the readings in vertical position first.

The position of the mercury meniscus was estimated twice by each observer, alternately; the readings were written on a small slip of paper and passed to a third party who entered the results on the record sheet, along with the time corresponding to each reading. This procedure was followed throughout, in the observations in ice as well as in steam, in order that each observer might be entirely

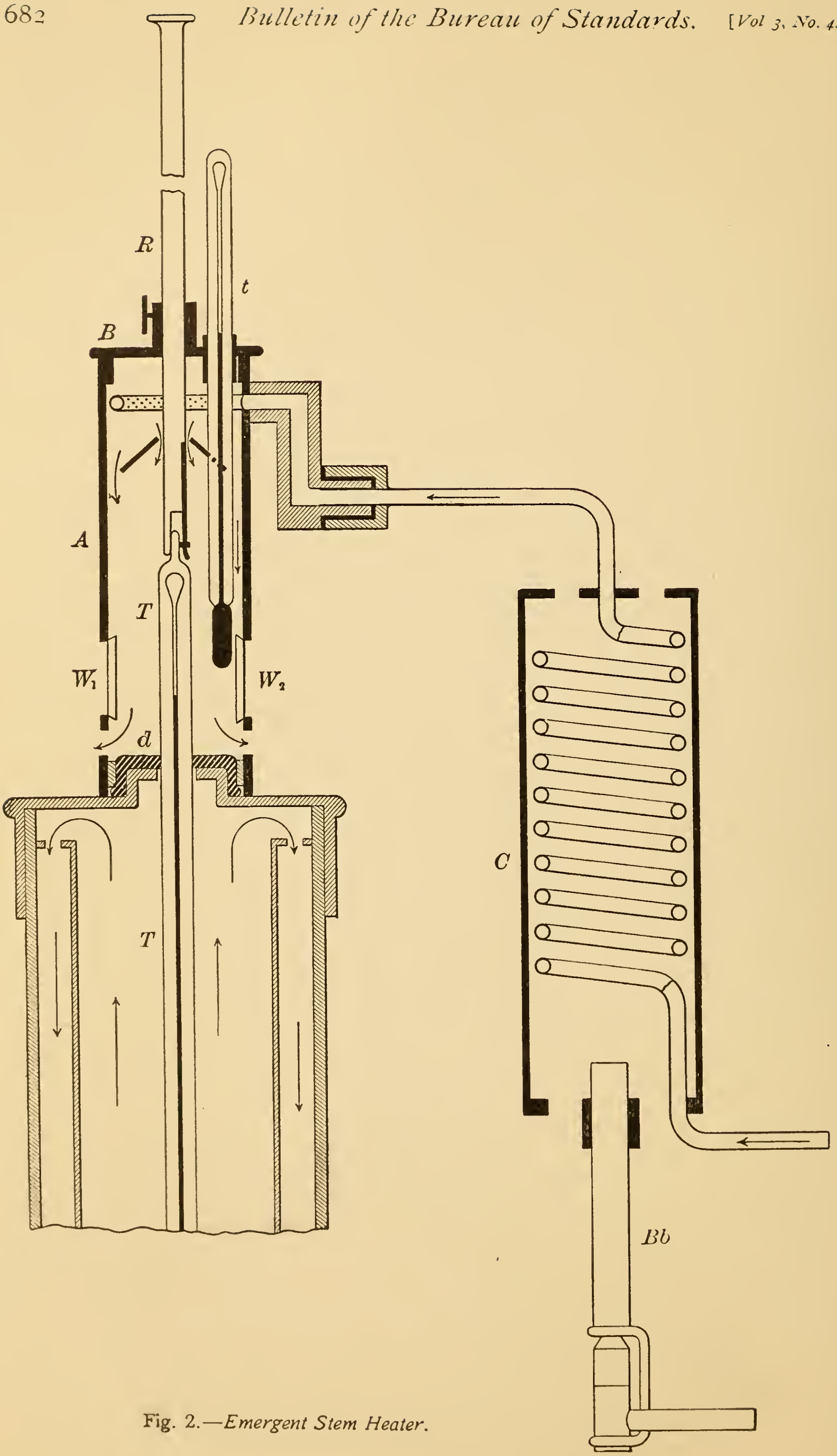

Fig. 2.—*Emergent Stem Heater.*

free from the inevitable influence exerted by a knowledge of the other's reading. The mean of the estimates by the two observers only very rarely differed by as much as 0°.003. In order to check the possible development of a systematic error in estimation, frequently throughout the work, micrometric determinations accompanying the estimates were made, and the order of agreement was always found to be within 0°.001 or 0°.002.

The temperature of the steam, corresponding to observed readings of the thermometers, was deduced from readings of the standard barometer.

Barometers.—While the observations in steam were being made, two other observers were making a series of observations on the Feuss primary standard barometer, which is shown removed from its usual wall mounting, in Fig. 3 (on the left). In this barometer, which consists of a combination of the cistern and siphon types, the difference in height of the mercury in the two glass tubes of equal diameter (14 mm) is measured by means of a vernier, graduated to 0.02 mm, which slides over a silvered brass tube, inclosing the barometer tubes, and on which the scale is engraved. The temperature of the mercury column is given by a thermometer, mounted within and read through a slot in the brass tube. The position of the mercury meniscus could be varied by means of the capstan wheel at the lower end of the barometer, so that, knowing the volume of the space above the mercury in the closed limb, it was possible to determine the pressure of the residual gas in this space. This "correction for residual gas" was determined at frequent intervals throughout the work. The corrections to the scale readings of the thermometer attached to the barometer have been determined at different times. By comparison with a meter bar, the corrections to which were known, it was found that the errors of graduation on the scale of the barometer were less than 0.02 mm. The errors of reading with this barometer were within 0.02 or 0.03 mm for different observers. Each determination of the barometric pressure was accompanied by a measurement of the height of the upper and lower meniscus. The observed barometer heights were reduced[25] to 0° and

[25] By means of the table given in Guillaume's Thermometrie de Precision, p. 323.

latitude 45°, sea level.[26] Corrections were also applied for the slight amount of residual gas in the closed limb of the barometer, and for the differential pressure of the upper and lower menisci. From a time chart of reduced barometric pressures, the pressure corresponding to the time of observation on the standard thermometer in steam was found, and from this pressure the corresponding temperature of the steam.[27]

Some of the earlier determinations of the fundamental intervals apparently showed a systematic departure from the values given in the certificates and found at the International Bureau. This discrepancy at once suggested a possible difference in the barometric measurements and led to an extended investigation of the barometer. After a redetermination of the scale errors, residual gas correction, and correction to attached thermometer of the Feuss barometer, it was compared with two laboratory barometers of the Fortin type, made by H. J. Green, of Brooklyn, N. Y. The order of agreement between these three barometers was within 0.1 mm, the limit of accuracy to which the Green barometers could be used. The desirability of a direct comparison with another standard barometer led to the construction of the barometer shown in Fig. 3. This consists of a U tube, siphon type, barometer, 21 mm internal diameter. It was filled after being kept exhausted for some days by a mercury pump. The positions of the menisci could be varied by running mercury into or out of the open limb of the barometer by means of the simple device shown in the illustration, so that the correction for residual gas could be determined. The distance between the upper and lower menisci was measured by means of a Société Genevoise cathetometer and a standard bronze meter bar, graduated into millimeters on a silver strip in the neutral axis of the bar. This meter bar, together with the table of corrections to its scale, was kindly placed at our disposal by Mr. L. A. Fischer, of the Division of Weights and Measures.

[26] By means of the equation giving the acceleration of gravity at any latitude L and elevation A (in meters) above sea level, in terms of the acceleration at latitude 45°, sea level, as computed by Broch (Trav. et Mem. du Bur. Int. 1, p. A9; 1881)

$$\frac{g_{L,A}}{g_{45,0}}=(1-0.00259\cos 2L)\,(1-0.000000196A).$$

[27] From the tables given by Broch (loc. cit.), based on a recomputation of the experiments of Regnault (see also Guillaume, Thermometrie de Precision, pp. 115 and 327).

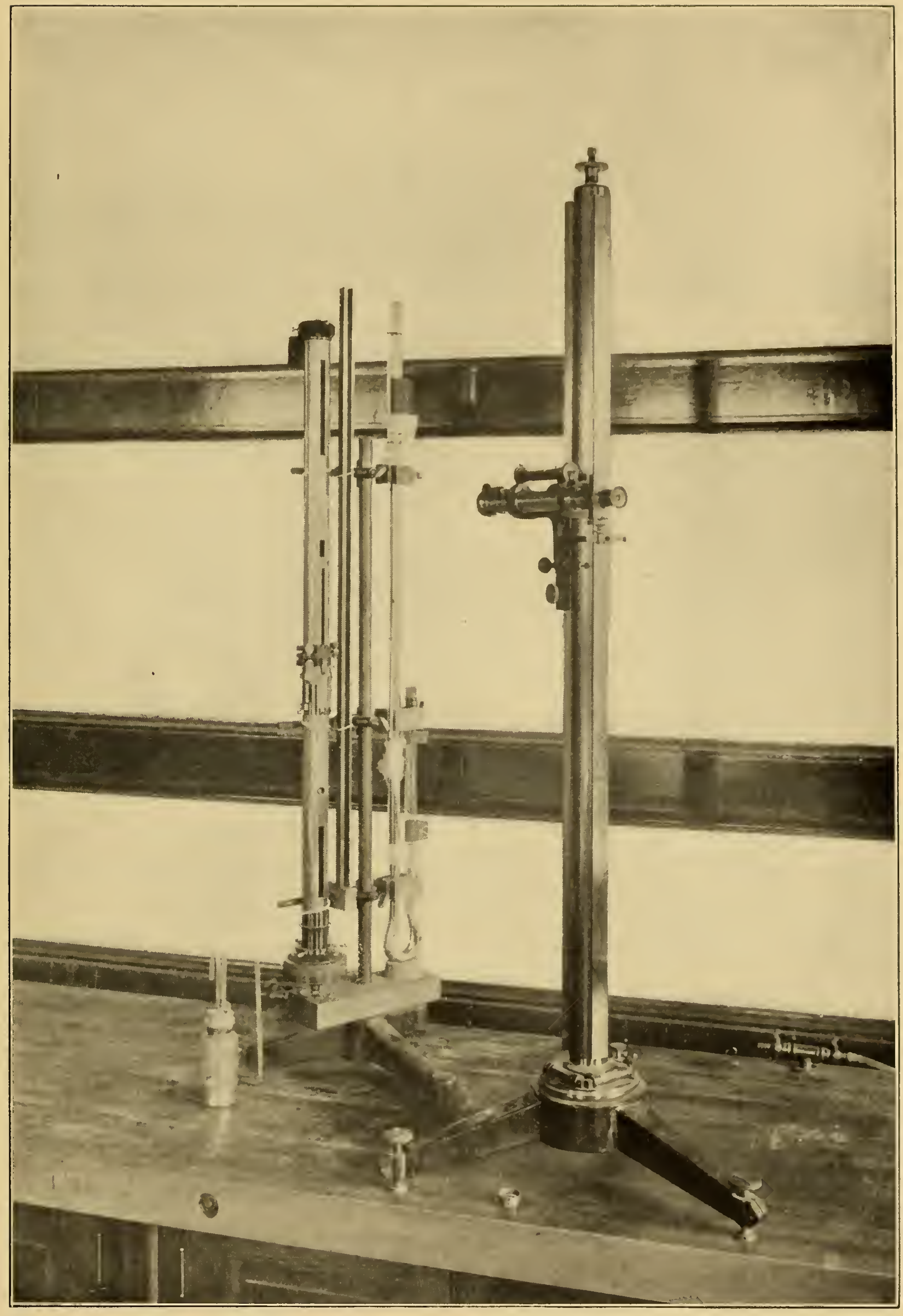

Fig. 3.—*Standard Barometers.*

The position of the top of the meniscus was sharply defined by surrounding the barometer tube with a black paper cylinder, the lower edge of which was brought almost down to the plane of the meniscus, and using an illuminated white background. This method effectively cuts off stray reflections so that settings of the cross hair of the telescope could be readily made on the plane of the upper surface of the meniscus to within 0.02 or 0.03 mm. The temperature of the mercury column was measured by two thermometers with bulbs in contact with the barometer tube and protected against external influences by wrapping with cotton.

The illustration, Fig. 3, shows (from right to left) the standard barometer, the standard meter, and the Feuss barometer, and nearby the cathetometer. These barometers have been intercompared

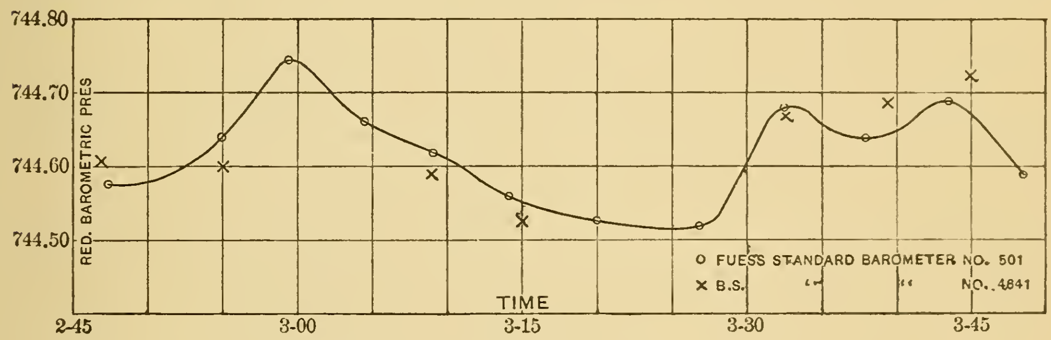

Fig. 4.—*Intercomparison of Barometers.*

many times and no certain difference as great as 0.02 mm has been found. The accompanying curve, Fig. 4, will serve as an illustration of the intercomparisons.

After the completion of these intercomparisons an opportunity presented itself for a comparison of the standard Feuss barometer with another standard barometer by the same maker; the difference between the two instruments being less than 0.02 mm as given by the mean of four intercomparisons.

Observations in ice.—Immediately after completion of each observation in steam, the thermometer was removed from the steam bath, rapidly cooled to 50° by stroking with a slightly moistened cloth, and then plunged into the super-cooler for a period of about 15 seconds, where it was cooled to a temperature of —2° or —3°, when it was rapidly transferred to the ice bath. Observations, consisting of

alternate readings (divisions front and back) by each observer, were in general begun within 1½ to 2 minutes after the thermometer was removed from the steam bath, and were completed at the end of about 4 or 5 minutes, where eye estimates alone were made, and in about 6 minutes where micrometer measurements were made in addition. When the thermometer was withdrawn from steam, the recorder started a stop watch, and in this way entered with each observation the time that had elapsed since the thermometer was removed from the steam.

The zero begins to recover (rise) as soon as the thermometer is removed from steam; the true ice-point reading corresponding to the temperature of the steam could only be obtained by instantly cooling the thermometer from 100° down to 0° and reading before any appreciable recovery had taken place, or, knowing the rate of recovery, by applying a correction to find the reading corresponding to the time when the thermometer was removed from the steam. Under practical working conditions the ice-point determination can be begun within 2 and completed in about 4 minutes. We have therefore reduced all our ice-point determinations to a time of 3 minutes after removal from steam, using as the rate of recovery per minute 0°.0011. The time required for a thermometer to attain its lowest reading in the ice bath can be greatly reduced by the use of a small stirrer made of thin glass rod, the circular arc of the stirrer surrounding the bulb of the thermometer. This procedure was followed in the latter half of the work.

The observations were made in the Ice-Point Apparatus, shown in Fig. 1, which consists of two concentric glass vessels surrounded by the polished metal cylinder shown to the right of and removed from the apparatus. The two glass vessels are filled with finely divided ice, the ice in the inner vessel being thoroughly saturated with distilled water. The latter precaution is very important, as otherwise very significant errors may arise entirely aside from the degree of purity of the ice used.[28] The ice was heaped up around the thermometer, as shown in the illustration, and a narrow deep channel was made to permit of reading. The thermometer was held in position by an adjustable spring, which pressed it against two

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[28] Pernet: Trav. et Mem. du Bur. Int., 1, p. B. 12, 1881.

V-shaped rests. A brass tube, with a fine wire suspension running through it and carrying a plumb bob at the bottom, of the same diameter as the tube, permitted of rapid adjustment of the thermometer supports in a vertical position. The telescope was adjusted to horizontal position by means of a striding level.

Throughout the work every precaution was taken to insure the purity of the ice. The ice used was a commercial artificial product made from filtered and twice-distilled water, and was very clear and pure. The ice itself and everything brought into contact with it was thoroughly washed with distilled water. After every series of determinations the resistances of several samples of water drawn from the ice bath were measured by means of the Kohlrausch Conductivity Bridge and Nernst Electrolytic Cell, shown in Fig. 1. The specific resistance after a complete series of ice-point determinations (7 thermometers) was in nearly every instance almost or quite as high as the specific resistance of the distilled water furnished by a block-tin condenser, and the resistance of the water obtained by melting the ice directly was considerably higher.

In the usual method of making the observations in ice the reading is obtained under the condition of falling meniscus, to avoid which the thermometer was supercooled to -2° or -3°, as mentioned above, before putting it into the ice.[29] The *Supercooler* used for this purpose, shown on the tripod stand in Fig. 1, consisted of a bottle, well covered with felt, containing a suitable mixture of salt and ice, into which dipped a test tube containing sufficient mercury to cover the bulbs of the thermometers.

Sample F. I. determination.—A complete series of observations and computations included in two determinations of the fundamental interval of Tonnelot No. 4332 are given in Table II. Under the columns W and D are given the independent readings by two observers, the two numbers included in each bracket being readings with divisions in front of and with divisions back of the mercury thread, respectively. The temperature of the steam was obtained from a time chart of reduced barometric pressures, after adding the small excess of pressure given by the water manometer, in the

[29] The authors have since found that a similar procedure was used by Grützmacher; Abhandl. der Phys.-Tech. Reichsanstalt, 3, p. 252, 1900.

manner already indicated. The times given under "Observations in Ice" refer to the time that has elapsed since the thermometer was removed from steam. $Z_{3'}$ is the ice point reading reduced to a time of 3 minutes after removal from steam.

TABLE II.

Fundamental Interval Determination.

Nov. 10, 1906. Tonnelot No. 4332.

Observations in Steam										Observations in Ice			
Horizontal					Vertical								
Time	W	Time	D	Man.	Time	W	Time	D	Man.	Time	W	Time	D
11:21:00	99.641	11:20:00	99.638	2.0	11:24:30	99.577	11:23:30	99.583	2.6	2:15	−.095	2:40	−.090
	.642		.637	1.8		.577		.582	1.8		−.092		−.090
11:22:30	99.641	11:22:00	99.639	2.1	11:26:00	99.579	11:25:10	99.580	2.3	3:15	−.091	3:30	−.086
	.638		.638	2.0		.580		.580	2.2		−.088		−.092
2:59:00	99.595	2:58:00	99.596	2.0	3:01:45	99.542	3:01:00	99.543	3.0	2:40	−.096	2:05	−.094
	.598		.597	2.0		.548		.545	3.0		−.091		−.088
3:00:00	99.598	2:59:15	99.596	2.1	3:03:45	99.538	3:02:00	99.544	2.6	3:30	−.094	2:55	−.087
	.597		.597	2.2		.547		.544	2.8		−.085		−.094
11:21:45	99.640	11:21:00	99.638	2.0	11:25:15	99.578	11:24:20	99.581	2.2	2:45	−.091	3:05	−.090
Cal. cor.	−.010		−.010			−.011		−.011			.000		.000
E. P. cor.	+.001		+.001			+.001		+.001			+.001		+.001
I. P. cor.	.000		.000			+.056		+.056			+.007		+.007
Z. cor.	+.083		+.082			+.083		+.082					
	99.714		99.711			99.707		99.709			−.083		−.082
Steam T. =	99.625		99.625			99.624		99.625		$Z_3'=$	−.083	$Z_3'=$	−.082
F. I. =	100.089		100.086			100.083		100.084					
2:58:30	99.597	2:59:37	99.596	2.1	3:02:45	99.544	3:01:30	99.544	2.8	3:05	−.091	2:30	.091
Cal. cor.	−.011		−.011			−.012		−.012			.000		.000
E. P. cor.	+.001		+.001			+.001		+.001			+.001		+.001
I. P. cor.	.000		.000			+.056		+.056			+.007		+.007
Z. cor.	+.083		+.082			+.083		+.082					
	99.670		99.668			99.672		99.671			−.083		−.083
Steam T. =	99.580		99.580			99.581		99.582		$Z_3'=$	−.083	$Z_3'=$	−.082
	100.090		100.088			100.091		100.089					

Results of fundamental interval determinations.—The fundamental intervals of all the thermometers were determined after the conclusion of the intercomparisons, in the manner already described. The results of the several series of determinations carried out at different times are given in Tables III–V. In Table VI are given the mean value of the fundamental interval of each thermometer as found by the authors, the value found at the Bureau International, and the final value adopted. Two additional series of 18 determinations of the fundamental interval of Tonnelot 11801 have given 99°.9986. The observations on Baudin 16017 were interrupted at the conclusions of the series on November 20 on account of the appearance of a small crack in the glass in the neighborhood of the auxiliary reservoir at 50°. An independent series of 18 observations taken on this thermometer at an earlier date, along with Tonnelot 11801, gave 100°.0046, which is practically identical with the mean of the 8 determinations (100°.0049) given in Table V.

The determinations here summarized show that the average difference (irrespective of sign) between the independent but simultaneous determinations by the two observers is 0°.002, the values found by one observer being quite consistently higher throughout by about 0°.001. In only 11 of the 216 determinations do the two observers differ by as much as 0°.005, yet it will be seen that the range throughout which the fundamental intervals of the thermometers varies at different times is about 0°.015. The authors are of the opinion that these variations are due in part to a sticking of the meniscus and resulting variations in capillary pressure, as the observations in both steam and ice are taken with a practically static meniscus, and in part to slight variations in the coefficient of expansion of the glass, depending on the state of strains present. The average difference between the fundamental intervals found by the authors and the values given in the certificates of the International

TABLE III.

Fundamental Interval Determinations.

Date	Observer	Thermometers No. 4331	4332	4334	4335	4336	4623	4624
1905								
Oct. 23	W.		100.071	100.072	100.090	100.082	100.061	100.073
	D.		.067	.073	.086	.080	.064	.068
"	W.	100.067	100.079	100.076	100.089	100.088	100.067	100.079
	D.	.062	.075	.076	.089	.088	.067	.079
Oct. 24	W.	100.066	100.076	100.076	100.083	100.083	100.061	100.073
	D.	.063	.076	.073	.082	.081	.061	.072
"	W.	100.072	100.087	100.072	100.090	100.093	100.065	100.073
	D.	.069	.083	.071	.088	.084	.065	.073
"	W.	100.074	100.083	100.071	100.096	100.093	100.070	100.073
	D.	.071	.084	.075	.093	.093	.070	.074
"	W.	100.080	100.085	100.069	100.094	100.091	100.072	100.080
	D.	.083	.084	.074	.094	.088	.072	.079
Nov. 6	W.	100.071	100.084	100.073	100.088	100.089	100.066	100.072
	D.	.068	.083	.072	.088	.086	.067	.073
"	W.	100.064	100.078	100.072	100.086	100.079	100.061	100.071
	D.	.059	.076	.072	.087	.079	.059	.071
Nov. 10	W.	100.069	100.079	100.073	100.084	100.088	100.066	100.068
	D.	.073	.078	.071	.084	.088	.064	.066
"	W.	100.066	100.082	100.074	100.081	100.089	100.061	100.066
	D.	.066	.081	.073	.081	.085	.063	.065
Mean F. I.		100.0690	100.0796	100.0729	100.0876	100.0863	100.0651	100.0724

TABLE IV.

Fundamental Interval Determinations.

Date	Observer	Thermometers No.						
		4331	4332	4334	4335	4336	4623	4624
1906								
Nov. 6	W.	100.071	100.073	100.063	100.087			
	D.	.071	.072	.064	.087			
Nov. 8	W.	100.076	100.084	100.076	100.089	100.083		
	D.	.072	.081	.080	.089	.084		
"	W.	100.069	100.077	100.073	100.091	100.082	100.062	100.073
	D.	.064	.075	.066	.088	.084	.065	.072
Nov. 10	W.	100.063	100.086	100.076	100.089	100.087	100.073	100.076
	D.	.063	.085	.076	.087	.086	.071	.077
"	W.		100.082	100.069	100.095	100.090	100.066	100.075
	D.		.081	.069	.095	.086	.065	.076
Nov. 12	W.		100.076	100.065	100.087	100.089	100.067	100.072
	D.		.073	.063	.088	.085	.065	.072
"	W.		100.080	100.071	100.083	100.083	100.064	100.074
	D.		.076	.071	.081	.081	.062	.075
Nov. 13	W.		100.075	100.070	100.087	100.083	100.065	100.072
	D.		.076	.069	.088	.085	.061	.070
"	W.		100.076	100.071	100.082	100.092	100.076	100.069
	D.		.074	.069	.078	.088	.072	.066
"	W.		100.080	100.071	100.081	100.086	100.065	100.071
	D.		.080	.069	.082	.084	.062	.071
"	W.		100.081	100.075	100.092	100.090	100.078	100.080
	D.		.080	.072	.089	.089	.077	.078
Mean F. I.		100.0686	100.0783	100.0704	100.0870	100.0853	100.0676	100.0733

TABLE V.

Fundamental Interval Determinations.

Date	Ob-server	Thermometers No.						
		15962	15282	15555	15583	16016	16017	16018
1906								
Nov. 19	W.	99.915	99.895	100.020	99.997	100.028	100.010	100.000
	D.	.914	.897	.020	100.000	.026	.007	99.999
"	W.	99.908	99.896	100.015	99.995	100.026	100.007	99.997
	D.	.904	.895	.015	100.003	.026	.007	.998
Nov. 20	W.	99.920	99.899	100.022	99.996	100.026	100.002	100.004
	D.	.920	.899	.020	.997	.025	.000	.002
"	W.	99.913	99.893	100.009	99.991	100.021	100.003	99.996
	D.	.909	.890	.008	.991	.018	.003	.996
Nov. 22	W.	99.913	99.893	100.008	99.993	100.022		99.995
	D.	.911	.894	.004	.990	.019		.996
Nov. 23	W.	99.917	99.893	100.020	100.001	100.020		99.998
	D.	.914	.890	.022	99.999	.017		100.000
"	W.	99.913	99.894	100.018	99.999	100.024		100.001
	D.	.911	.894	.016	100.001	.023		99.994
Nov. 26	W.	99.915	99.894	100.019	100.009	100.026		99.995
	D.	.912	.890	.020	.009	.026		.995
Nov. 27	W.	99.912	99.884	100.018	99.992	100.018		99.991
	D.	.910	.885	.021	.991	.019		.990
Nov. 28	W.	99.919	99.895	100.016	99.988	100.020		99.995
	D.	.915	.894	.021	.990	.017		.995
Nov. 30	W.	99.912	99.891	100.017	99.993	100.026		99.993
	D.	.908	.889	.016	.991	.022		.996
"	W.	99.913	99.884	100.016	99.986	100.021		100.002
	D.	.910	.889	.015	.986	.020		99.998
Dec. 1	W.	99.909	99.892	100.013	99.983	100.020		99.994
	D.	.907	.891	.010	.984	.017		.991
Mean F. I.		99.9124	99.8922	100.0161	99.9944	100.0220	100.0049	99.9966

TABLE VI.

Final Résumé of Fundamental Intervals

Thermometer	Fund. Int. found by authors	Fund. Int. given in certifs. of Bur. Int.	Fund. Int. adopted
Tonnelot No. 4331	100.0688	100.0632	100.0660
" 4332	100.0789	100.0760	100.0774
" 4334	100.0716	100.0800	100.0716
" 4335	100.0873	100.0896	100.0884
" 4336	100.0858	100.0844	100.0851
" 4623	100.0664	100.0668	100.0666
" 4624	100.0729	100.0710	100.0720
" 11801	99.9986	99.9968	99.9977
Baudin No. 15962	99.9124	99.9071	99.9098
" 15963	(*)	100.0856	100.0856
" 15282	99.8922	99.8937	99.8930
" 15555	100.0161	100.0199	100.0180
" 15583	99.9944	99.9890	99.9917
" 16016	100.0220	100.0171	100.0196
" 16017	100.0049	99.9945	99.9997
" 16018	99.9966	99.9912	99.9939

*Thermometer No. 15963 broke during the intercomparisons, and before the beginning of the fundamental interval determinations.

Bureau is $+0^{\circ}.002$. In two instances the differences attain nearly $0^{\circ}.010$, namely, for Tonnelot 4334 and Baudin 16017. On account of the very satisfactory agreement in the two series of determinations with Tonnelot 4334, taken about a year apart, the mean value given by these two series has been adopted as the final value; in all other instances the final value adopted is the mean of the determinations by the authors and the values found at the International Bureau. When the crack appeared in the neighborhood of the auxiliary reservoir of 16017 it seemed that the difference between the values found by the authors and those found at the International Bureau might possibly be explained by a change in the volume of the auxiliary reservoir, due to strains which were evidently present there. A redetermination of the calibration corrections at 50°, however, gave the identical value given in the certificate.

By a correlation of the data contained in the present paper and some subsequent intercomparisons of small groups of these thermometers, it seems highly probable that the fundamental intervals of some of the thermometers at the close of this work are slightly different from the values at the beginning. It would have been better if some of the fundamental interval determinations had been carried out previous to as well as at the conclusion of the intercomparisons. In general an interval of some days between successive determinations is preferable.

Emergent stem and distillation corrections.—The amount by which the readings of thermometers in steam are lowered, due to the emergent stem above the rubber diaphragm, was determined by means of a Mahlke[30] "thread thermometer," the bulb of which consisted of a capillary tube 102 mm long, and stem of finer bore. Micrometric measurements gave the mean temperature of the equivalent length of stem (102 mm, which corresponds to from 13° to 18° on the standard thermometers) to about 0°.01, which corresponds to only a few ten-thousands of a degree in the total stem correction. The values of the stem correction found in this way for a thermometer, for which the length of 1° = 7 mm is given in Table VII.

TABLE VII.

Corrections for Emergent Stem.

n No. of degrees emergent	S. C. Stem Correction Observed	S. C. Stem Correction Computed	Obs.—Comp.
−0°.3	0°.0002		
+ .3	.0013	0°.0016	−0°.0003
.3	.0018	.0016	+ .0002
.7	.0033	.0034	− .0001
.7	.0036	.0034	+ .0002
1.05	.0053	.0052	+ .0001
1.45	.0078	.0075	+ .0003
1 45	.0081	.0075	+ .0006
2.15	.0134	.0125	+ .0009
2.90	.0192	.0190	+ .0002

[30] Mahlke, Zs. für Instrumentenkunde, **13**, p. 58; 1893.

The measurements are expressed, to a sufficient degree of accuracy, by the equation

$$\text{Stem correction} = 0^\circ\!.0006 + 0.0032n + 0.00108n^2$$

where $n=$ number of degrees emergent above the sheet-rubber diaphragm. For the standard thermometers used the length of 1° ranges from about 6 to 8 mm; with 1° emergent the stem correction for these degree lengths varies only between 0°.0050 and 0°.0053. If there is 0°.5 emergent, the mean temperature of the last degree is about 85°.

Two independent series of observations, micrometer and estimated, on Baudin No. 15555 in steam, with 0°.3 emergent, gave as the rate of distillation of the mercury 0°.0006 per minute. Accompanying observations of the ice point gave an identical result. For the observations in steam it generally took from 5 to 6 minutes for a complete series of measurements (two estimates alternately by each observer) in horizontal and vertical positions, with divisions front and back. The total distillation of mercury during this time was therefore about 0°.0035. The ice point determined immediately after was therefore also lowered by this amount, so that the resulting fundamental interval is too high by about 0°.0017. Now the stem correction under these conditions is 0°.0016. It will be seen therefore that the usual procedure in the determination of the fundamental interval, viz, reading the thermometer with about 0°.3 emergent, gives the same value for the F. I. as is obtained by avoiding (or correcting for) both the stem correction and the distillation off the end of the thread. There still remains however the advantage in avoiding distillation that the relative changes of the ice and steam points in a series of determinations may be compared.

Zero depression and recovery.—Experiments on the depression of the ice point of thermometers made of French *verre dur*, Jena 16^{III} normal, and Jena 59^{III} borosilicate glasses have been made by several investigators.[31]

[31] Guillaume: Trav. et Mem. du Bur. Int., 5, p. 49; 1886. Böttcher: Zs. für Instrumentenkunde, 8, p. 409; 1888.

The results of the experiments of Guillaume for *verre dur* in the interval 0° to 100° are represented by the equation—

$$Z_0 - Z_t = 0.000\,888\,6t + 0.000\,001\,084t^2,$$

where Z_0 and Z_t are the corrected ice-point readings of the thermometer after 0° and after t°, respectively.

Thiesen, Scheel, and Sell[32] find for *verre dur*—

$$Z_0 - Z_t = 0.001\,003\,6t + 0.000\,000\,928t^2$$

A recomputation of their experiments by Scheel, omitting some of the more uncertain observations near 0° led to the equation—

$$Z_0 - Z_t = 0.001\,199t - 0.000\,000\,52t^2$$

In the present investigation the thermometers were placed in ice and kept at approximately 0° for a period of 4 to 8 weeks preceding each series of intercomparisons and fundamental interval determinations.[33] When removed from the ice box care was taken to prevent any appreciable rise of temperature until the ice-point reading was taken; the thermometers were then quickly transferred to the comparator and after the intercomparisons at 10°, 20°, etc., the corresponding ice-point readings were taken. This gave the necessary data for the determination of the relation between the ice-point reading and the temperature to which the thermometer had been exposed. The results obtained in this way are summarized in Table VIII. The values given in the table, corresponding to any temperature, are the mean values for all the thermometers, taken several times, so that they represent the mean of a large number of ice-point observations.

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[32] Thiesen, Scheel, and Sell: Zs. für Instrumentenkunde, **16**, p. 58; 1896. Scheel: Zs. für Instrumentenkunde, **17**, Beiblatt No. 13, p 98, 1897.

[33] In some instances an exposure of 4 to 6 weeks in ice, after the thermometers had been heated to 100°, seemed insufficient to bring the glass to an equilibrium condition corresponding to 0°, while at other times under apparently similar conditions a considerably shorter exposure seemed sufficient.

TABLE VIII.

Zero Depressions.

t° Temperature	$Z_0 - Z_t$ Observed Depression	$Z_0 - Z_t$ Comp. Depression	Obs.-Comp.
10°	0°.0099	0°.0094	+0°.0005
20°	.0186	.0191	− .0005
30°	.0295	.0291	+ .0004
40°	.0390	.0393	− .0003
50°	.0498	.0497	+ .0001
60°	.0599	.0605	− .0006
70°	.0723	.0715	+ .0008
80°	.0825	.0827	− .0002
90°	.0944	.0942	+ .0002
100°	.1063	.1060	+ .0003

The above observations satisfy the equation—

$$Z_0 - Z_t = 0.000\ 930t + 0.000\ 001\ 300t^2$$

The ice-point readings correspond to the observed readings taken immediately after each temperature in question and reduced to a time of 3 minutes after the thermometer was removed from the bath.

For the purposes of comparison the depression curves for *verre dur* found by Guillaume, by Thiesen, Scheel, and Sell, and by the authors are given in Fig. 5.

The observations in ice gave 0°.0015 as the recovery per minute, at 0°, 3 minutes after the thermometer is removed from the bath, and 0°.0011 as the recovery corresponding to 4 minutes. Guillaume has found that the rate of recovery during the first few minutes in ice is practically independent of the temperature to which the thermometer has just previously been exposed. This is in agreement with our observations, as will be seen from the accompanying table obtained from the ice-point determinations of one of the series of intercomparisons.

TABLE IX.

Rate of Zero Recovery.

Temperature	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	Mean
Rate of recovery after 3 minutes............	.0004	.0016	.0013	.0018	.0018	.0015	.0011	.0012	.0018	.0018	0.0015

After 10° the rate of recovery is the same to within the limits of error of experiments of this kind.

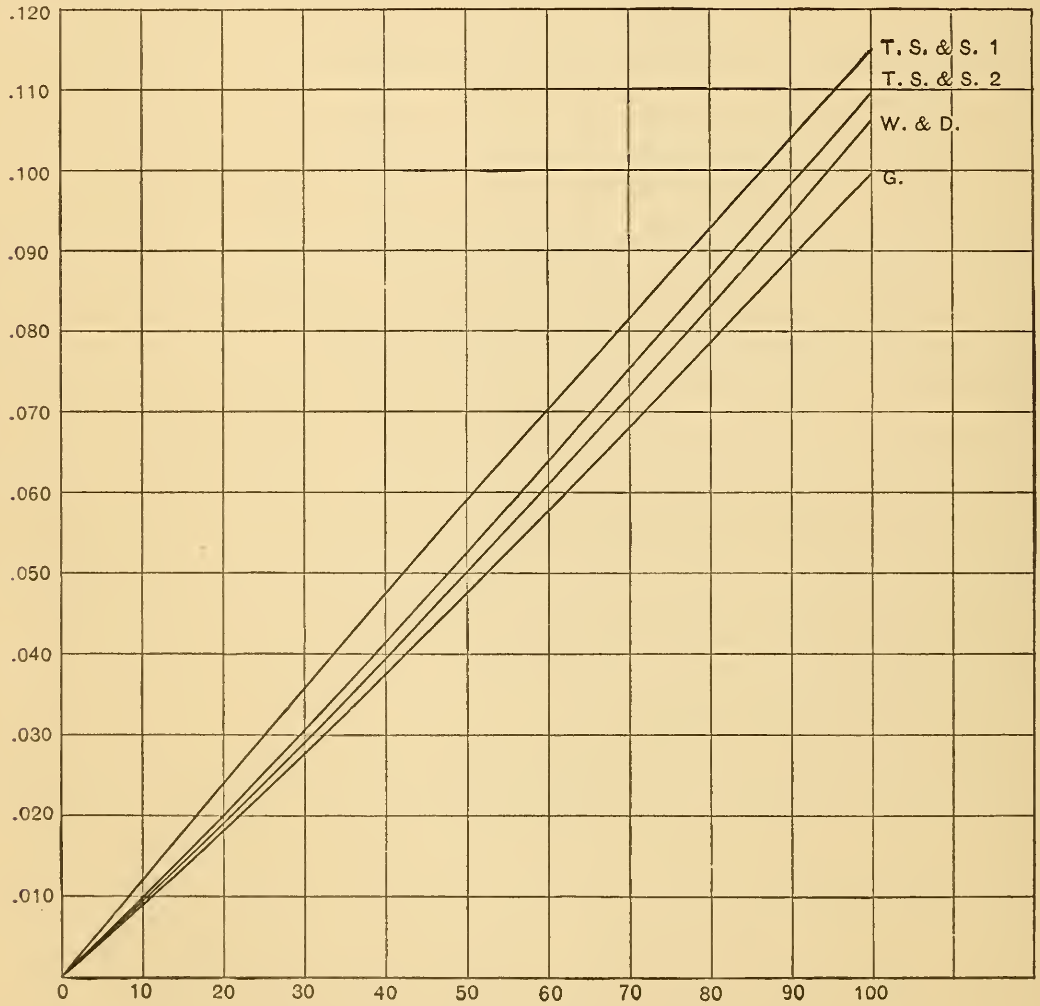

Fig. 5.—*Depression of Zero for Verre Dur.*

IV. INTERCOMPARISON OF THERMOMETERS.

The intercomparisons of the sixteen primary standard thermometers available for this investigation were carried out in a specially designed *Comparator* which has admirably fulfilled every require-

ment of this work, as well as that of testing the large number of laboratory and special thermometers that are submitted to the Bureau each year for comparison with its standards.

Comparator.—The comparator, shown diagrammatically in section in Fig. 6, and by photographic reproduction in Fig. 7, is a triple wall water bath, which can be used in either horizontal or vertical position, with water coil for rapid heating and cooling and electric heating coil for precise temperature regulation, and with a rapid circulation and thorough stirring of the water produced by an electrically driven propeller. The upper part of the tank is made of three concentric brass tubes and contains within the inner one a holder with places for fourteen thermometers, as ordinarily used in testing, two primary standards and twelve thermometers to be compared. Between the outer two of these tubes is a packing of hair felt, and between the inner two the downward circulation of water which comes upward around the thermometers in the inner space. Plate glass windows, W, W, 60 cm long and about 8 cm wide, are let into opposite sides to permit of reading the thermometers. These windows, being opposite each other, afford excellent direct illumination, especially when an incandescent lamp, with diffusing screen interposed, is placed in front of the window away from the observer.

The thermometers are read by means of the microscope, T, moved by rack and pinion, on a vertical support, in front of the window. The thermometer holder, H, may be conveniently rotated to bring the thermometers successively into the field of the microscope by means of a belt running from the pulley, P, in the water-tight cover, over idlers, down to a hand wheel, placed on the side of the tank, which is operated by the right hand of the observer, while he grasps the hand wheel of the telescope rack and pinion movement with his left hand, and thus rapidly raises or lowers the telescope to bring the mercury meniscus of the thermometer into the center of the field.

The construction of the holder, H, will be understood from Figs. 6 and 7. The thermometers are pressed outward against V-shaped rests and held in position by the phosphor bronze springs, s, s. The holder, partly loaded with thermometers, is shown alongside the comparator.

The lower portion of the comparator consists of an inner brass chamber covered with a heavy layer of felt and a removable outer

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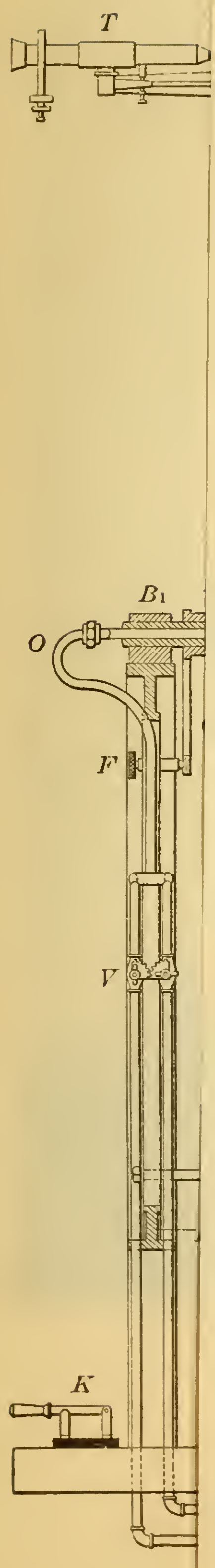
brass casing. In the inner chamber are contained the means for securing temperature regulation and circulation of water. First, there is a three-eighths inch copper coil, W, making sixteen turns, with inlet, I, and outlet, O, through the shaft on which the comparator is supported. Through this coil either hot or cold water may be circulated. For temperatures below that of the room, water from an ice-water tank is used; at higher than room temperatures hot water is used only for rapid heating, and for exact regulation the electric heating coil, E, furnishes the energy. This coil, which is of a type made by the Simplex Electric Heater Company for the trade, was found to fulfill the requirements of this apparatus very satisfactorily. It consists of a three-eighths inch brass tube, in which are inserted two asbestos covered coils, combinations of which give three rates of heating for any E. M. F., and which, in conjunction with the rheostat, R, permit regulation from a few watts to 800 watts. Stirring is effected by means of a propeller, S, driven by the 75-watt motor, M_1, belted to a wheel, P_2, on the shaft of the comparator, from which another belt runs over idlers, P_3, to the pulley, P_4, on the propeller shaft, which runs through the stuffing box, G. The object of this arrangement was to permit of one of the most convenient features of the comparator, i. e., its adaptation to horizontal as well as vertical comparison. By drawing the spring key, F, tipping the comparator to a horizontal position and releasing the key, the comparison of a set of thermometers may be continued in horizontal position without any change whatever in the working of any part of the comparator.

At temperatures below that of the room it is important, in order to secure satisfactory temperature regulation, that the cold water be delivered at the inlet, I, at a constant temperature, which would not be the case if there were a standpipe connecting the comparator to the cold-water reservoir. To secure this condition a constant stream of cold water is kept circulating to the comparator through the pipe marked Ice Water Pipe, past the inlet I, and back to the cold-water reservoir (or to drain) through the pipe marked Return Pipe, the amount of resistance to the flow of water in this circuit, and therefore the effective pressure available at the inlet I being controlled by the valve V_4. This circulation is maintained by means of a small centrifugal pump, C, driven by a 75-watt motor, M_2. From



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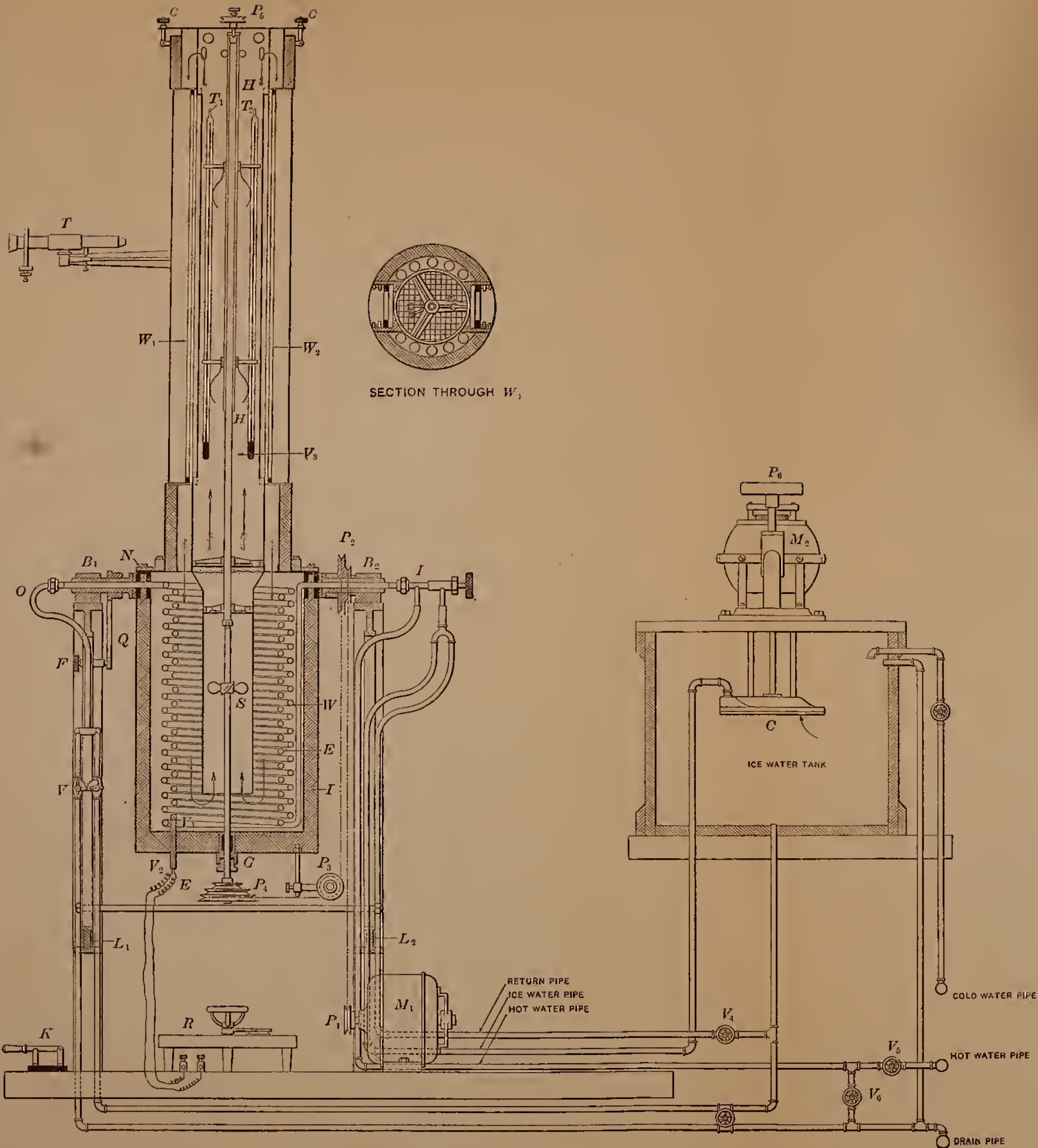

Fig. 6.—*Thermometer Comparator.*

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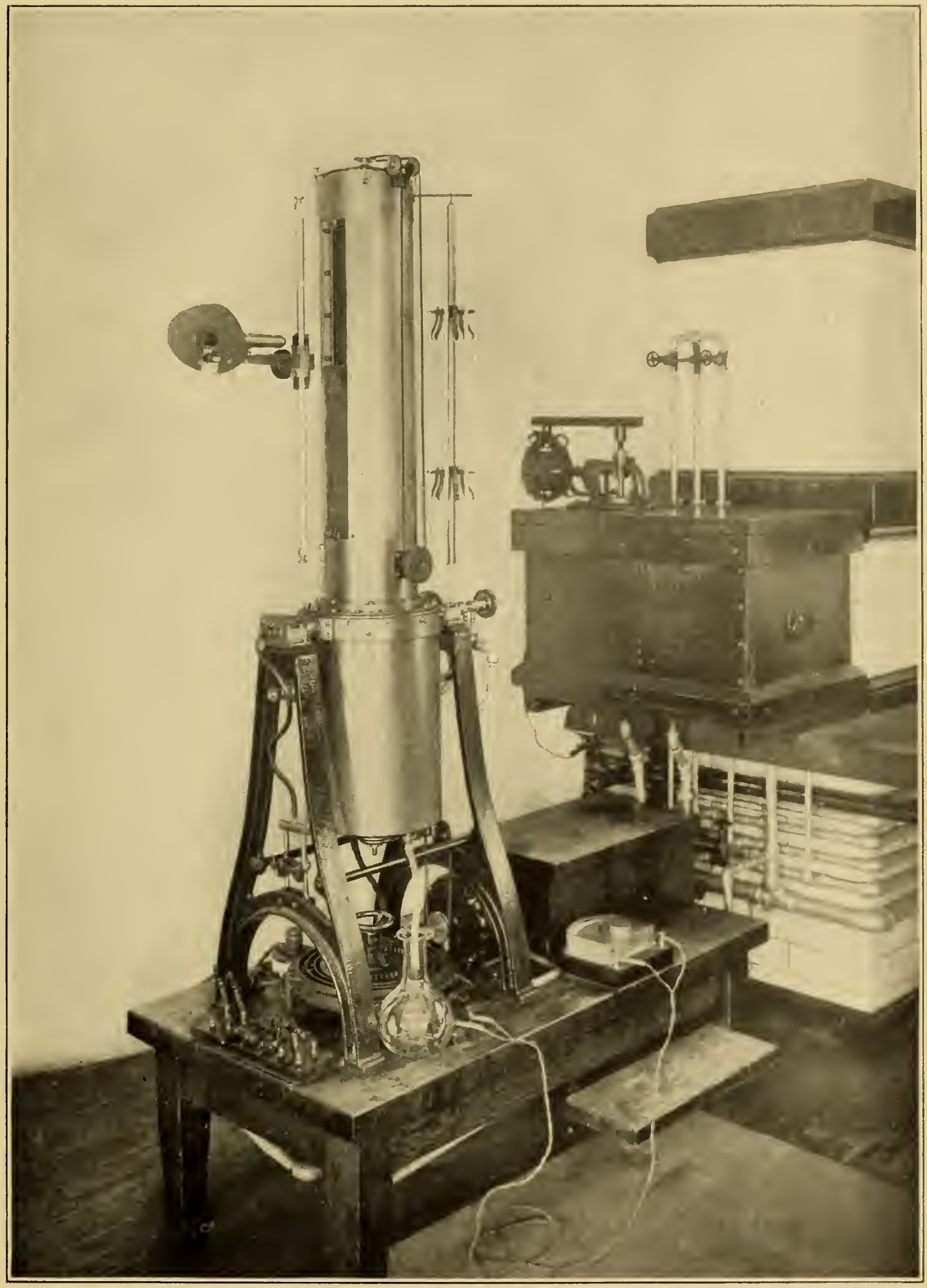

Fig. 7.—*Thermometer Comparator.*

this cold-water circulation the requisite amount of water to maintain the desired temperature is admitted through a sensitive stopcock, I, into the copper coil W, after passing through which it may either be returned to the cold-water reservoir or passed to the sewer by turning the geared valves V.

Hot water, drawn from the hot-water distribution system in the laboratory, for rapidly raising the temperature from one test point to another, is admitted into the coil W through the valve V_5. A constant circulation of the hot water, and thus a steady temperature condition, is provided for by the valve V_6.

With this comparator and equipment and with reasonable constancy in the temperature of the room, any temperature within the range 5° to 90° may be quickly attained at the rate of 2° to 4° per minute, and may be held constant to within a few thousandths of a degree for many minutes or made to vary at any desired rate.

Method of Intercomparison.—The intercomparisons were made at every 10°, in the interval 0° to 100°, in nine series, with the thermometers mounted in the holder of the comparator in the order indicated in Table X.

Before each series of intercomparisons the thermometers were kept in ice for a period of 4 to 8 weeks.

TABLE X.

Series of Intercomparisons.

Series No.	Thermometers Intercompared							Range
I	4331	4332	4334	4335	4336	4623	4624	0°–50°
II	11801	15962	15963	15282	15555	15583		0°–50°
III	4331	4332	4334	4335	4336	4623	4624	0°–50°
IV	11801	15962	15963	15282	15555	15583		0°–50°
V	11801	15962	15963	4331	4332	4623		0°–50°
VI	11801	15962	15963	15282	16016	16017	16018	50°–100°
VII	11801	15962	15555	4331	4332	4623		0°–50°
VIII	11801	15962	15282	16016	16017	16018		50°–100°
IX	15962	15282	16016	16017	16018			50°–100°

In each series six or seven thermometers were intercompared at one time. This was done for several reasons, not alone on account of the large number of thermometers involved which would have greatly extended the time required for intercomparisons in groups of two or three thermometers, but also because of the important advantage that in reading this number of thermometers the observer does not keep in mind the last reading of any particular thermometer and is thus free from the unconscious but ever present influence of his last reading. The advantages of working with only two or three thermometers, viz, that the effect of irregularities in the rate of rise of temperature of the bath is of less importance and that the meniscus of the thermometers can more easily be prevented from disappearing behind a graduation mark in the course of a set of readings, are much minimized by the construction of the comparator. First, because of the extremely sensitive temperature control made possible by the electric heating and the constant and thorough stirring, a very uniform rise of temperature is readily obtainable, and if the thermometers are read at equal time intervals forward and backward, the first source of error is extremely small; and second, if in the course of a set of readings the meniscus of one or more of the thermometers should come behind a graduation mark the observer may turn the thermometer to the right or left, so that the meniscus may be viewed off the edge of the graduation or the telescope can be raised or lowered and the meniscus viewed from above and from below the graduation mark, when it is easy to estimate from the observed motion of the telescope whether the meniscus is above or below the center of the graduation. As the thermometers were turned in succession into the field of view of the telescope they occupied identical positions in the comparator while being read, so that the effect of any small constant difference of temperature in different parts of the section of the tank in which the bulbs are found does not enter.[34] It was therefore deemed permissible to mount the thermometers in the holder in the same serial order in the several series where the same thermometers were intercompared, as this was a great convenience in comparing two series and in checking the corrections that had been taken from the tables.

[34] Some experiments made during the work showed that the maximum differences in temperature between the top and bottom of the space inclosing the thermometers was less than 0°.001.

The comparator was so adjusted that the thermometers were held in a vertical position. The line of collimation of the reading telescope on the comparator was maintained horizontal with the aid of a striding level. To eliminate any outstanding errors of parallax the thermometers were always read with divisions before, then with divisions back, and again with divisions before the mercury thread. The curvature produced in the thermometers by the slight spring pressure of the holder[35] was entirely too small to give rise to any appreciable error.

The supply of electrical energy, or of cold water, to the comparator was carefully adjusted before every set of observations to give a slowly rising temperature, as readings with falling meniscus are liable to very considerable error. Indeed in those sets of observations, where the temperature was apparently constant or the rise was very small, the irregularities were nearly always greater than for a more rapid rise. The rate of rise used at different times varied from 0°005 to 0°009 per minute.

One of the most serious sources of error in mercurial thermometers, and which more than any other factor limits the precision attainable (now that glasses with small zero depression are available, and that methods of using thermometers have been found which almost completely eliminate the effects of thermal hysteresis) arises from the irregular variations of the capillary forces of the mercury meniscus, which produce a variable internal pressure, equivalent to some centimeters of mercury, depending on the form of the meniscus, the angle of contact, and the varying dimensions of and condition of surface of the capillary tube. These variable forces, which may give rise to irregularities amounting to many thousandths of a degree, manifest themselves in the calibration of thermometers, where it is frequently observed that the differential pressure exerted by the two menisci of the calibrating thread is sufficient to displace or hold in equilibrium considerable lengths of thread. Pernet[36] has shown that for a thermometer of very fine bore the difference between the indications for rising and falling temperatures may

[35] A holder has since been constructed in which the pressure of the springs against the thermometer is directly in line with the V-shaped supports against which the thermometer is held.

[36] Pernet: Zs. für Instrumentenkunde, 6, p. 382; 1886.

attain 0°07 C. It is frequently observed that one thermometer may indicate an absolutely constant temperature while those on either side of it, in the same bath, show a very decided rise, amounting to 0°01 or more. Micrometric measurements of the position of the meniscus are, therefore, not to be recommended, as the order of precision of estimation for a trained observer considerably exceeds the accuracy to which a single indication of the thermometers can be relied upon. As has been previously stated, a large number of micrometric and estimated readings has shown that the two observers rarely differed by as much as 0°003. It was, therefore, deemed best to dispense with time-consuming micrometric measurements, and to obtain a larger number of estimated readings, to better eliminate the irregularities in the behavior of the thermometers.

The thermometers were read at intervals of 10 seconds forward and backward, until each had been read six times, when a similar set of observations were made by the other observer, beginning at the other end of the set. The thermometers were then turned in the holder so that the divisions were back of the mercury thread, when two similar sets of six readings each were made alternately by each observer, after which the thermometers were again turned to their former position with divisions before the mercury thread, and another set of readings taken by each observer. At every intercomparison temperature each thermometer was therefore read 24 times by each observer.

Observations in ice.—Before each series of intercomparisons the thermometers were kept at 0° for a period of 4 to 8 weeks, and the ice-point reading corresponding to long-continued exposure at 0° (Z_0) was determined.

Immediately after completion of the intercomparisons at any temperature, the zero readings corresponding to that temperature were determined in melting ice. In these determinations the thermometer was quickly transferred from the comparator to the supercooler (see p. 687) and then to the ice-point apparatus. Two independent estimates were made by each observer. The mean time of reading was generally very near to 3 minutes after removal from the comparison bath. The zero was always reduced to that corresponding to 3 minutes after removal from the bath, by making proper allowance for the zero recovery as previously described.

After completion of the observations in ice the thermometer was returned to the comparison bath, the temperature of which all the while had been maintained constant. After all the thermometers had been observed in ice the series of ice-point observations was repeated. In all comparisons below room temperature care was taken that the thermometer was not heated appreciably above the temperature of the comparison bath in the process of transference from the bath to the supercooler. After the completion of the series of intercomparisons (at the highest temperature) the thermometers were exposed to a temperature of 100° (steam) for a period of about 20 minutes and the ice-point readings corresponding to 100° (Z_{100}) determined. All the precautions previously described were taken to insure the purity of the ice used in these determinations.

The observed ice-point readings were then corrected for calibration, external pressure,[37] internal pressure, and fundamental interval. These corrected ice-point readings,

$$Z_0 \quad Z_{10} \quad Z_{20} \quad \ldots\ldots \quad Z_{100}$$

corresponding 0°, 10°, 20°, etc., were then plotted, which gave a complete *depression curve* for that series of intercomparisons. From this depression curve the *mean* Z_0 was computed and thus the reduced zeros corresponding to 10°, 20°, 30°, etc. It was found that the ice-point readings reduced in this way gave more concordant results than the use of the ice point determined only once after the particular temperature in question. The ice point is one of the most difficult points to determine. It is possible to multiply readings indefinitely at any temperature in the comparison bath, but the accuracy gained in this way is lost unless the ice point is determined with equal accuracy. We have, therefore, adopted the practice, even if a single temperature only is to be accurately measured, of determining the ice point at a number of temperatures and deducing the *mean zero* corresponding to that temperature from the observed depression curve. The *mean corrected ice-point* reading correspond-

[37] Including pressure of column of water of height equal to the distance from center of bulb to the zero mark as well as excess (or deficiency) of reduced barometric pressure over (or below) standard atmospheric pressure (760 mm).

ing to each temperature, reduced in the way indicated above, was then applied, with its sign changed, as the *zero correction* to the observed reading of the thermometer.

Sample intercomparison.—The laboratory record of the observations and reductions of an intercomparison of six thermometers at one temperature (80°) is reproduced, as an illustration, in Tables XI and XII, the asterisk (*) indicating the first thermometer read in each group of observations. The mean of the 24 observed readings of each thermometer is corrected for calibration, external[38] and internal pressure, zero, and fundamental interval. The temperatures thus found are temperatures on the *verre dur scales* defined by the several thermometers. In the last line of each table are given the *supercorrections* resulting from the intercomparisons, i. e., the correction that must be applied to the scale defined by each thermometer at the given temperature to reduce to temperature on the mean scale defined by all of the thermometers included in the intercomparison.

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[38] Including pressure of column of water in the comparator above the center of the bulb of thermometer as well as the excess (or deficiency) of the reduced barometric pressure above (or below) standard atmospheric pressure.

TABLE XI.

Intercomparison Record.

Observer, C. W. W. Computations by C. W. W. Checked by H. C. D.

Position of Therm.	Tonnelot No. 11801	Baudin No. 15962	Baudin No. 15282	Baudin No. 16016	Baudin No. 16017	Baudin No. 16018
Divs. Before	79.806	79.886	79.899	79.973	79.952	80.051*
	.809	.893	.904	.983	.962	.067
	.828	.900	.916	.996	.970	.067
	.832	.901	.928	80.000	.985	.085
	.845	.913	.941	.005	.992	.087
	.848	.922	.946	.015	80.000	.100
	79.8280	79.9025	79.9223	79.9953	79.9968	80.0762
Divs. Back	*79.850	79.922	79.950	80.023	80.003	80.102
	.873	.942	.958	.035	.007	.102
	.872	.945	.962	.040	.020	.118
	.892	.951	.974	.046	.026	.118
	.893	.953	.986	.051	.040	.134
	.900	.969	.994	.056	.042	.136
	79.8800	79.9470	79.9707	80.0418	80.0230	80.1183
	79.798	79.860	79.895	79.959	79.946	80.042*
	.798	.867	.896	.967	.949	.045
	.803	.886	.901	.969	.950	.046
	.805	.884	.902	.978	.951	.051
	.811	.888	.909	.983	.956	.053
	.813	.894	.911	.992	.962	.061
	79.8047	79.8798	79.9023	79.9747	79.9523	80.0497
Divs. Before	*79.805	79.890	79.905	79.988	79.963	80.065
	.830	.900	.918	.997	.974	.068
	.835	.902	.937	80.002	.993	.093
	.852	.929	.946	.015	.996	.095
	.852	.929	.952	.028	80.007	.107
	.880	.950	.962	.044	.011	.108
	79.8423	79.9167	79.9367	80.0123	79.9907	80.0893
Mean Obs. Rdgs	**79.8388**	**79.9115**	**79.9330**	**80.0060**	**79.9857**	**80.0834**
Calibration Cor	+.0746	—.0219	—.0750	+.0160	+.0159	—.0683
Ext. Pres. Cor	—.0052	—.0056	—.0069	—.0054	—.0057	—.0055
Int. Pres. Cor	+.0691	+.0737	+.0918	+.0467	+.0486	+.0468
Zero Cor	+.0228	—.0335	—.0256	—.0505	—.0527	—.0657
Cor. to F. I	+.0011	+.0721	+.0856	—.0157	+.0002	+.0048
Temp. Verre dur Scale	**80.0012**	**79.9963**	**80.0029**	**79.9970**	**79.9920**	**79.9955**
Cor. to Mean Scale	**—.0037**	**+.0012**	**—.0054**	**+.0005**	**+.0055**	**+.0020**

TABLE XII.

Intercomparison Record.

Observer H. C. D. Computations by H. C. D. Checked by C. W. W.

Position of Therm.	Tonnelot No. 11801	Baudin No. 15962	Baudin No. 15282	Baudin No. 16016	Baudin No. 16017	Baudin No. 16018
Divs. Before	*79.752	79.824	79.848	79.923	79.902	80.003
	.752	.840	.856	.928	.904	.003
	.763	.843	.860	.944	.918	.020
	.780	.853	.871	.946	.921	.020
	.783	.855	.880	.956	.939	.040
	.799	.868	.895	.957	.940	.041
	79.7713	79.8472	79.8683	79.9423	79.9207	80.0212
Divs. Back	79.806	79.874	79.900	79.970	79.951	80.048*
	.806	.875	.900	.972	.954	.050
	.813	.894	.909	.980	.956	.051
	.815	.895	.910	.986	.964	.059
	.833	.900	.922	.994	.966	.061
	.834	.903	.927	80.000	.978	.074
	79.8195	79.8902	79.9113	79.9837	79.9615	80.0572
	*79.770	79.846	79.862	79.938	79.916	80.014
	.777	.850	.868	.944	.920	.016
	.778	.850	.873	.948	.924	.022
	.788	.856	.881	.955	.930	.025
	.788	.856	.883	.953	.931	.033
	.795	.858	.885	.957	.936	.030
	79.7823	79.8527	79.8753	79.9492	79.9262	80.0233
Divs. Before	79.754	79.823	79.850	79.916	79.898	80.000*
	.754	.826	.852	.923	.904	.003
	.763	.838	.855	.927	.905	.004
	.764	.844	.860	.942	.921	.024
	.783	.852	.872	.946	.925	.023
	.784	.855	.878	.952	.942	.042
	79.7670	79.8397	79.8612	79.9343	79.9158	80.0160
Mean Obs. Rdgs	**79.7850**	**79.8575**	**79.8790**	**79.9524**	**79.9311**	**80.0294**
Calibration Cor	+.0752	−.0215	−.0750	+.0160	+.0159	−.0683
Ext. Pres. Cor	−.0052	−.0056	−.0069	−.0054	−.0057	−.0055
Int. Pres. Cor	+.0691	+.0737	+.0917	+.0467	+.0486	+.0467
Zero Cor	+.0228	−.0335	−.0256	−.0505	−.0527	−.0657
Cor. to F. I.	+.0011	+.0721	+.0856	−.0157	−.0002	+.0048
Temp. Verre dur Scale	**79.9480**	**79.9427**	**79.9488**	**79.9435**	**79.9374**	**79.9414**
Cor. to Mean Scale	**−.0044**	**+.0009**	**−.0052**	**+.0001**	**+.0062**	**+.0022**

Results of intercomparisons.—The results of the nine series of intercomparisons and the additional intercomparison of the seven Tonnelot thermometers at 20° are given in Tables XIII–XVII.

The temperatures there given are the final reduced temperatures on the *verre dur scale* of each thermometer. Each temperature was found in the manner previously explained and illustrated in Tables XI and XII. The two nearly equal temperatures recorded for each intercomparison point are the results of independent intercomparisons by each observer. The supercorrections to the mean scale of the thermometers included in each separate intercomparison are given below each temperature in smaller print.

TABLE XIII.

Intercomparisons—Series I and III.

Tonnelot No. 4331	Tonnelot No. 4332	Tonnelot No. 4334	Tonnelot No. 4335	Tonnelot No. 4336	Tonnelot No. 4623	Tonnelot No. 4624
10.0768 +.0041	10.0801 +.0008	10.0850 −.0041	10.0776 +.0033	10.0817 −.0008	10.0850 −.0041	10.0804 +.0005
10.0936 +.0003	10.0897 +.0042	10.0987 −.0048	10.0905 +.0034	10.0928 +.0011	10.0945 −.0006	10.0972 −.0033
20.0586 +.0037	20.0633 −.0010	20.0677 −.0054	20.0640 −.0017	20.0605 +.0018	20.0628 −.0015	20.0592 +.0031
20.1025 +.0041	20.1066 .0000	20.1110 −.0044	20.1093 −.0027	20.1051 +.0015	20.1078 −.0012	20.1038 +.0028
30.0405 −.0005	30.0371 +.0029	30.0432 −.0032	30.0396 +.0004	30.0389 +.0011	30.0433 −.0033	30.0375 +.0025
30.0444 −.0006	30.0434 +.0004	30.0480 −.0042	30.0437 +.0001	30.0400 +.0038	30.0461 −.0023	30.0409 +.0029
40.0692 +.0007	40.0684 +.0015	40.0731 −.0032	40.0704 −.0005	40.0703 −.0004	40.0708 −.0009	40.0673 +.0026
40.1036 +.0011	40.1029 +.0018	40.1086 −.0039	40.1050 −.0003	40.1049 −.0002	40.1084 −.0037	40.0994 +.0053
50.0548 +.0005	50.0553 .0000	50.0553 .0000	50.0531 +.0022	50.0557 −.0004	50.0593 −.0040	50.0537 +.0016
50.1049 +.0008	50.1058 −.0001	50.1043 +.0014	50.1038 +.0019	50.1071 −.0014	50.1106 −.0049	50.1035 +.0022
10.1201 +.0004	10.1173 +.0032	10.1264 −.0059	10.1173 +.0032	10.1194 +.0011	10.1189 +.0016	10.1238 −.0033
10.1756 −.0017	10.1709 +.0030	10.1781 −.0042	10.1703 +.0036	10.1733 +.0006	10.1730 +.0009	10.1761 −.0022
20.0634 −.0057	20.0541 +.0036	20.0594 −.0017	20.0575 +.0002	20.0520 +.0057	20.0602 −.0025	20.0574 +.0003
20.0900 −.0034	20.0861 +.0005	20.0881 −.0015	20.0862 +.0004	20.0808 +.0058	20.0896 −.0030	20.0852 +.0014
30.0544 −.0025	30.0485 +.0034	30.0553 −.0034	30.0508 +.0011	30.0474 +.0045	30.0554 −.0035	30.0512 +.0007
30.1268 −.0046	30.1198 +.0024	30.1256 −.0034	30.1212 +.0010	30.1155 +.0067	30.1254 −.0032	30.1213 +.0009
40.1406 −.0002	40.1400 +.0004	40.1429 −.0025	40.1404 .0000	40.1349 +.0055	40.1454 −.0050	40.1386 +.0018
40.0913 −.0013	40.0909 −.0009	40.0929 −.0029	40.0896 +.0004	40.0854 +.0046	40.0921 −.0021	40.0877 +.0023
50.0783 +.0011	50.0796 −.0002	50.0777 +.0017	50.0820 −.0026	50.0798 −.0004	50.0828 −.0034	50.0754 +.0040
50.0379 +.0020	50.0419 −.0020	50.0413 −.0014	50.0412 −.0013	50.0385 +.0014	50.0422 −.0023	50.0365 +.0034

TABLE XIV.

Intercomparisons—Series II and IV.

Tonnelot No. 11801	Baudin No. 15962	Baudin No. 15963	Baudin No. 15282	Baudin No. 15555	Baudin No. 15583
9.8199 −.0062	9.8082 +.0055	9.8133 +.0004	9.8134 +.0003	9.8102 +.0035	9.8171 −.0034
9.9072 −.0061	9.8964 +.0047	9.9003 +.0008	9.9002 +.0009	9.8967 +.0044	9.9057 −.0046
19.9965 −.0097	19.9794 +.0074	19.9854 +.0014	19.9872 −.0004	19.9799 +.0069	19.9926 −.0058
20.0641 −.0100	20.0473 +.0068	20.0534 +.0007	20.0545 −.0004	20.0470 +.0071	20.0583 −.0042
29.9679 −.0088	29.9560 +.0031	29.9566 +.0025	29.9575 +.0016	29.9522 +.0069	29.9642 −.0051
30.0183 −.0099	30.0049 +.0035	30.0059 +.0025	30.0081 +.0003	30.0024 +.0060	30.0111 −.0027
39.9317 −.0095	39.9153 +.0069	39.9171 +.0051	39.9258 −.0036	39.9158 +.0064	39.9276 −.0054
39.9499 −.0084	39.9376 +.0039	39.9374 +.0041	39.9454 −.0039	39.9335 +.0080	39.9454 −.0039
49.9921 −.0076	49.9770 +.0075	49.9803 +.0042	49.9897 −.0052	49.9809 +.0036	49.9872 −.0027
50.0247 −.0075	50.0129 +.0043	50.0110 +.0062	50.0200 −.0028	50.0132 +.0040	50.0214 −.0042
9.9038 −.0057	9.8934 +.0047	9.8943 +.0038	9.8980 +.0001	9.8957 +.0024	9.9034 −.0053
9.9587 −.0042	9.9514 +.0031	9.9519 +.0026	9.9534 +.0011	9.9518 +.0027	9.9600 −.0055
20.0367 −.0074	20.0253 +.0040	20.0267 +.0026	20.0277 +.0016	20.0234 +.0059	20.0361 −.0068
20.0470 −.0036	20.0384 +.0050	20.0406 +.0028	20.0441 −.0007	20.0389 +.0045	20.0514 −.0080
29.9452 −.0064	29.9327 +.0061	29.9365 +.0023	29.9361 +.0027	29.9346 +.0042	29.9478 −.0090
29.9743 −.0051	29.9645 +.0047	29.9688 +.0004	29.9664 +.0018	29.9640 +.0052	29.9774 −.0082
40.0185 −.0062	40.0057 +.0066	40.0158 −.0035	40.0105 +.0018	40.0031 +.0092	40.0201 −.0078
40.0896 −.0063	40.0769 +.0064	40.0868 −.0035	40.0808 +.0025	40.0760 +.0073	40.0899 −.0066
49.9764 −.0011	49.9683 +.0070	49.9847 −.0094	49.9770 −.0017	49.9662 +.0091	49.9794 −.0041
50.0224 −.0002	50.0155 +.0067	50.0289 −.0067	50.0232 −.0010	50.0175 +.0047	50.0258 −.0036

TABLE XV.

Intercomparisons—Series V and VII.

Tonnelot No. 11801	Baudin No. 15962	Baudin No. 15963	Tonnelot No. 4331	Tonnelot No. 4332	Tonnelot No. 4623
9.9845 −.0024	9.9756 +.0065	9.9818 +.0003	9.9819 +.0002	9.9818 +.0003	9.9870 −.0049
10.0388 −.0015	10.0295 +.0078	10.0384 −.0011	10.0414 −.0041	10.0351 +.0022	10.0405 −.0032
19.9396 +.0020	19.9319 +.0097	19.9419 −.0003	19.9442 −.0026	19.9447 −.0031	19.9473 −.0057
20.0297 +.0013	20.0215 +.0095	20.0314 −.0004	20.0340 −.0030	20.0329 −.0019	20.0363 −.0053
29.9136 −.0014	29.9068 +.0054	29.9159 −.0037	29.9120 +.0002	29.9110 +.0012	29.9138 −.0016
29.9425 +.0029	29.9369 +.0085	29.9513 −.0059	29.9484 −.0030	29.9450 +.0004	29.9485 −.0031
39.9970 −.0021	39.9879 +.0070	39.9974 −.0025	39.9951 −.0002	39.9971 −.0022	39.9950 −.0001
40.0584 −.0004	40.0515 +.0065	40.0600 −.0020	40.0598 −.0018	40.0582 −.0002	40.0604 −.0024
50.0591 −.0009	50.0511 +.0071	50.0602 −.0020	50.0577 +.0005	50.0616 −.0034	50.0594 −.0012
50.0223 −.0024	50.0123 +.0076	50.0223 −.0024	50.0183 +.0016	50.0234 −.0035	50.0207 −.0008
10.0378 −.0054	10.0361 −.0037	10.0268 +.0056	10.0307 −.0017	10.0288 +.0036	10.0340 −.0016
9.9832 −.0064	9.9798 −.0030	9.9713 +.0055	9.9727 +.0041	9.9745 +.0023	9.9792 −.0024
19.9984 −.0023	19.9941 +.0020	19.9892 +.0069	19.9965 −.0004	19.9996 −.0035	19.9987 −.0026
19.9482 −.0032	19.9422 +.0028	19.9379 +.0071	19.9468 −.0018	19.9469 −.0019	19.9482 −.0032
29.9946 −.0016	29.9873 +.0057	29.9884 +.0046	29.9974 −.0044	29.9936 −.0006	29.9968 −.0038
30.0331 −.0006	30.0259 +.0066	30.0287 +.0038	30.0361 −.0036	30.0356 −.0031	30.0355 −.0030
40.0550 −.0026	40.0427 +.0097	40.0468 +.0058	40.0564 −.0040	40.0567 −.0043	40.0568 −.0044
40.0058 +.0003	39.9965 +.0096	40.0016 +.0045	40.0111 −.0050	40.0107 −.0046	40.0110 −.0049
50.0954 +.0009	50.0801 +.0162	50.0919 +.0044	50.1037 −.0074	50.1041 −.0078	50.1024 −.0061
50.0489 −.0009	50.0362 +.0118	50.0451 +.0029	50.0522 −.0042	50.0531 −.0051	50.0523 −.0043

TABLE XVI.

Intercomparisons—Series VI and VIII.

Tonnelot No. 11801	Baudin No. 15962	Baudin No. 15963	Baudin No. 15282	Baudin No. 16016	Baudin No. 16017	Baudin No. 16018
49.9902	49.9827	49.9943	49.9888	49.9818	49.9865	49.9829
−.0035	+.0040	−.0076	−.0021	+.0049	+.0002	+.0038
50.0424	50.0321	50.0431	50.0381	50.0305	50.0369	50.0324
−.0059	+.0044	−.0066	−.0016	+.0060	−.0004	+.0041
60.0058	59.9954	60.0060	60.0008	59.9899	59.9947	59.9927
−.0079	+.0025	−.0081	−.0029	+.0080	+.0032	+.0052
60.0500	60.0405	60.0517	60.0487	60.0353	60.0401	60.0399
−.0063	+.0032	−.0080	−.0050	+.0084	+.0036	+.0038
70.0037	69.9973	70.0087	70.0069	69.9953	69.9996	69.9978
−.0024	+.0040	−.0074	−.0056	+.0060	+.0017	+.0035
70.0096	70.0026	70.0142	70.0139	70.0013	70.0031	70.0021
−.0029	+.0041	−.0075	−.0072	+.0054	+.0036	+.0046
79.9123	79.9082	79.9151	79.9132	79.9050	79.9028	79.9063
−.0033	+.0008	−.0061	−.0042	+.0040	+.0062	+.0027
79.9657	79.9622	79.9673	79.9658	79.9610	79.9563	79.9583
−.0033	+.0002	−.0049	−.0034	+.0014	+.0061	+.0041
90.0734	90.0722	Broken.	90.0730	90.0725	90.0673	90.0674
−.0024	−.0012	....	−.0020	−.0015	+.0037	+.0036
90.1257	90.1244	....	90.1281	90.1249	90.1207	90.1207
−.0016	−.0003	....	−.0040	−.0008	+.0034	+.0034

Tonnelot No. 11801	Baudin No. 15962	Baudin No. 15282	Baudin No. 16016	Baudin No. 16017	Baudin No. 16018
50.1124	50.1012	50.1053	50.1010	50.1012	50.1016
−.0086	+.0026	−.0015	+.0028	+.0026	+.0022
50.0669	50.0579	50.0612	50.0559	50.0573	50.0577
−.0074	+.0016	−.0017	+.0036	+.0022	+.0018
60.0566	60.0461	60.0527	60.0487	60.0493	60.0516
−.0058	+.0047	−.0019	+.0021	+.0015	−.0008
59.9994	59.9906	59.9974	59.9914	59.9919	59.9947
−.0052	+.0036	−.0032	+.0028	+.0023	−.0005
70.0555	70.0444	70.0575	70.0492	70.0477	70.0509
−.0046	+.0065	−.0066	+.0017	+.0032	.0000
70.0227	70.0149	70.0262	70.0174	70.0158	70.0191
−.0033	+.0045	−.0068	+.0020	+.0036	+.0003
80.0012	79.9963	80.0030	79.9971	79.9920	79.9955
−.0037	+.0012	−.0055	+.0004	+.0055	+.0020
79.9480	79.9427	79.9488	79.9435	79.9374	79.9414
−.0044	+.0009	−.0052	+.0001	+.0062	+.0022
90.1057	90.1066	90.1080	90.1066	90.1012	90.1022
−.0007	−.0016	−.0030	−.0016	+.0038	+.0028
90.0795	90.0769	90.0818	90.0784	90.0726	90.0743
−.0023	+.0003	−.0046	−.0012	+.0046	+.0029

TABLE XVII.

Intercomparisons—Series IX and 20°.

Tonnelot No. 11801.	Baudin No. 15962	Baudin No. 15282	Baudin No. 16016	Baudin No. 16017	Baudin No. 16018
	70.0044 +.0018	70.0095 −.0033	70.0075 −.0013	70.0060 +.0002	70.0035 +.0027
	69.9732 −.0009	69.9756 −.0033	69.9715 +.0008	69.9715 +.0008	69.9695 +.0028
79.9151 −.0006	79.9108 +.0037	79.9197 −.0052	79.9153 −.0008	79.9122 +.0023	79.9139 +.0006
79.9896 −.0013	79.9826 +.0057	79.9936 −.0053	79.9901 −.0018	79.9858 +.0025	79.9882 +.0001
	80.0315 −.0014	80.0302 −.0001	80.0308 −.0007	80.0293 +.0008	80.0285 +.0016
	79.9697 −.0015	79.9686 −.0004	79.9691 −.0009	79.9669 +.0013	79.9667 +.0015
	90.0660 +.0009	90.0652 +.0017	90.0694 −.0025	90.0687 −.0018	90.0652 +.0017
	90.0078 +.0007	90.0070 +.0015	90.0114 −.0029	90.0095 −.0010	90.0067 +.0018

Tonnelot No. 4331	Tonnelot No. 4332	Tonnelot No. 4334	Tonnelot No. 4335	Tonnelot No. 4336	Tonnelot No. 4623	Tonnelot No. 4624
19.9706 +.0023	19.9723 +.0006	19.9765 −.0036	19.9704 +.0025	19.9764 −.0035	19.9790 −.0061	19.9648 +.0081
20.0407 +.0026	20.0421 +.0012	20.0484 −.0051	20.0410 +.0023	20.0467 −.0034	20.0478 −.0045	20.0364 +.0069

By applying to the results of these separate intercomparisons of various groups of thermometers, methods of computation analogous to those employed in an incomplete thermometer calibration, the *final supercorrections to the thermometers to reduce their corrected*[39] *readings to the mean scale of all the thermometers* included in the intercomparisons at the given temperature were found as given in Table XVIII.

[39] The reading found by applying to the mean of the observed readings corrections for calibration, external pressure, internal pressure, zero, and fundamental interval (the observed ice point reading itself being corrected in a similar way and the resulting corrected ice point reading being applied, with its sign changed, as the zero correction to the observed reading of the thermometer).

TABLE XVIII.

Supercorrections to Thermometers.

Unit = 0°.001.

	Tonnelot No. 4331	Tonnelot No. 4332	Tonnelot No. 4334	Tonnelot No. 4335	Tonnelot No. 4336	Tonnelot No. 4623	Tonnelot No. 4624	Tonnelot No. 11801
10°	+1	+2	−5	+3	0	−2	−2	−4
20°	−1	−1	−5	−2	0	−4	+2	−2
30°	−2	+1	−4	0	+3	−3	+1	−2
40°	−2	−1	−4	−1	+1	−3	+2	−2
50°	−3	−5	−3	−3	−4	−5	−1	−2
60°								−6
70°								−3
80°								−3
90°								−2

	Baudin No. 15962	Baudin No. 15963	Baudin No. 15282	Baudin No. 15555	Baudin No. 15583	Baudin No. 16016	Baudin No. 16017	Baudin No. 16018
10°	+3	+2	+1	+5	−4			
20°	+7	+2	+2	+7	−4			
30°	+6	−2	+2	+6	−6			
40°	+8	−1	+1	+8	−4			
50°	+8	−4	0	+5	−3	+8	+5	+7
60°	+4	−5	−3			+6	+3	+2
70°	+4	−4	−5			+3	+3	+3
80°	+2	−3	−4			+1	+4	+2
90°	0		−2			−2	+2	+3

The average difference between the supercorrections found in the different intercomparisons is 0°.0025; the average difference between the supercorrections found by the two observers in the same series of intercomparisons is 0°.001. The fact that the supercorrections found at different times differ considerably more than the average difference between the observations of the two observers is, no doubt,

in part due to small inconsistencies in the ice-point determinations, but also, to some extent at least, to the fact that a thermometer may define a slightly different scale at different times, depending on the strains present in the glass, due to its past thermal history, and which modify the coefficient of expansion of the glass. Further evidence in support of this view is furnished by the results of the determinations of the fundamental intervals, as previously discussed. The variations in the value of the supercorrections found in different series of intercomparisons sometimes considerably exceed the limits of experimental error, and can hardly be explained by variations in the ice-point determinations on different dates, for it was extremely rare that the independent ice-point observations by the two observers differed by as much as 0°.003. In view of the precautions taken it is reasonably certain that contamination of the ice could play no part; and furthermore the ice-point readings after each temperature were made in the same ice for the entire group of thermometers, so that all would have been affected nearly equally, due to this cause. Notwithstanding the agreement between the ice-point readings of the two observers, it is nevertheless true that this reading may still be in error, due to the sticking of the meniscus in the capillary tube. While we have sought to minimize this source of error by supercooling the thermometer several degrees below 0°C before plunging it into the ice bath, thus insuring a rising meniscus, by cautiously tapping the thermometer with the finger, and by again determining the zero for each temperature after returning the thermometer to the comparison bath, we nevertheless feel that the discrepancies sometimes observed between the results obtained on different days are partly due to this cause.

Several of the thermometers, after their calibration and the determination of their pressure coefficients and fundamental interval, were compared with the standards of the International Bureau under the supervision of Dr. Guillaume, before being delivered to the Bureau of Standards. The certificates accompanying the thermometers give the following details of these intercomparisons:

> Tonnelot No. 11801 was compared with standards Nos. 4327 and 4330 in February, 1896, at temperatures near to 10°, 30°, and 50°. Baudin No. 15962 was compared with standards Nos. 15201, 15273, 14674, and 14675 in October and

November, 1903, at temperatures near to 10°, 20°, 30°, 40°, and 50°. Baudin Nos. 16016, 16017, and 16018 were compared with standards Nos. 14674 and 14675 in October, 1903, at temperatures near to 50° and 60°. The comparisons were carried out in a horizontal position, with twenty readings of each thermometer, ten readings with divisions above and ten with divisions below the mercury thread. The zeros of all the thermometers were determined before and after each intercomparison temperature.

The results of these intercomparisons are given in Table XIX. In the columns B. I. are given the supercorrections to the several thermometers included in these intercomparisons, i. e., the corrections that must be applied to the temperature scales defined by them to reduce to temperatures on the mean verre dur scale of the International Bureau. For purposes of comparison, the supercorrections to the mean scale of the thermometers of the Bureau of Standards, as

TABLE XIX.

Supercorrections to Mean Verre Dur Scale and to Mean Scale of 16 Standards Intercompared.

Temp.	Baudin No. 15962		Baudin No. 16016		Baudin No. 16017		Baudin No. 16018		Tonnelot No. 1180	
	B. I.	B. S.	B. I.	B. S.	B. I.	B. S.	B. I.	B. S.	B. I.	B. S.
10°	+0.002	+0.003							−0.005	−0.004
20°	+ .007	+ .007								
30°	+ .005	+ .006							− .006	− .002
40°	+ .008	+ .008								
50°	+ .005	+ .008	+0.002	+0.008	+0.002	+0.005	+0.005	+0.007	− 008	− .002
60°			+ .008	+ .006	+ .007	+ .003	+ .003	+ .002		

found in the present investigation, are given in the colums B. S. An inspection of the results of these two independent series of intercomparisons shows that the resulting supercorrections are quite small, that they have the same sign in every instance, and that the agreement is at least as good as could be expected, the average difference being only about +0°.0014.

V. RÉSUMÉ AND CONCLUSIONS.

In view of the number of thermometers included in the present investigation and the order of agreement between the supercorrections to the mean scale of these thermometers and the supercorrections to the *mean verre dur scale* as found by the Bureau International, we believe that the conclusion is fully warranted that the standard scale of temperature of the Bureau of Standards, in the interval 0° to 100° C., as defined by these thermometers and the supercorrections resulting from this investigation, is in agreement with the Hydrogen Scale of Temperature of the Bureau International to within the limits of accuracy at present attainable in mercurial thermometry (which may be put at about 0°.002). This conclusion is further confirmed by the many subsequent intercomparisons of these thermometers, in small groups of three or four, which have been made incident to the work of testing thermometers submitted to the Bureau for standardization, and especially by the intercomparisons of a number of these standards with two sensitive platinum resistance thermometers, by Mr. E. F. Mueller and one of the authors,[40] who found for the transition temperature of sodium sulphate[41] 32°.484. The final result of Richards and Wells[42] was 32°.483, temperatures in both cases being expressed on the International Hydrogen Scale.

The small values of and the quite systematic variations in the supercorrections throughout the scale bear convincing evidence to the fact that the small differences between the scales defined by the thermometers are to a great extent at least true differences due to slight variations in the chemical and physical properties of the glass. The authors can not too strongly express their appreciation of the painstaking work that has been done on these thermometers by the International Bureau at different times over a period of twenty years, as is shown by the systematic correlation of the standardizations of these thermometers.

[40] Dickinson and Mueller: This Bulletin, **3**, p. 641; 1907.

[41] Kahlbaum sodium sulphate recrystallized five times; and sodium sulphate made from the carbonate and recrystallized five times.

[42] Richards and Wells: Proc. American Acad., **38**, p. 431; 1902.

The constants of the thermometers (calibration corrections, external and internal pressure coefficients, and fundamental interval), which were determined at various times from 1885 to 1903 by the International Bureau, were again redetermined by the authors. The changes in the calibration corrections, for the thermometers examined, were found to be within the limits of experimental error. The values of the pressure coefficients found by the authors differed in the average from the values found at the International Bureau by about 0.5 per cent (0°.0000006 per mm of mercury), and the values of the fundamental interval by 0°.002.

A comparator, adapted to the intercomparison of thermometers in vertical or horizontal position, was designed by the authors and constructed in the instrument shops of the Bureau, which gave, by means of electric heating and cold-water circulation, every desired refinement in temperature control and convenience and quickness in manipulation for work in the interval 5° to 95°. A primary standard barometer was constructed, with which was compared at frequent intervals the Fuess standard barometer used in the steam-point determinations.

The sixteen primary standard thermometers were intercompared in groups of six or seven thermometers at intervals of 10°, from 10° to 90°, in nine series. The supercorrections to reduce the scale defined by each of the thermometers to the mean scale defined by all are given in Table XVIII, page 715.

From the ice-point observations, made after the thermometers had been at 0° for some weeks preceding each series of intercomparisons, after each intercomparison temperature, and at the end of each series of intercomparisons after exposure to steam, the *depression curve* for the glass (*verre dur*) of which these thermometers are made was found to be

$$Z_0 - Z_t = 0.000\,930t + 0.000\,001\,300t^2$$

where Z_0 = ice-point reading corresponding to long-continued exposure at 0°, and Z_t = ice-point reading corresponding to t°. The depression curve for *verre dur* resulting from these experiments is intermediate between that found by Guillaume and by Thiesen, Scheel, and Sell, the depression corresponding to 100° ($Z_0 - Z_{100}$)

being 0°.106, the corresponding values of $Z_0 - Z_{100}$ found by the above observers being 0°.0997 and 0°.1147, respectively.

The rate of recovery of the zero was found to be 0°.0015, 3 minutes after, and 0°.0011 4 minutes after the thermometer was removed from the water bath. As has been previously found by Guillaume, the rate of recovery of the zero for the first few minutes after the thermometer is plunged into the ice bath is practically independent of the temperature to which it has just previously been exposed, at least for temperatures in the interval 20° to 100°.

A series of experiments were made to determine the magnitude of the possible sources of error arising from distillation of mercury from the end of the column and from the slight cooling of the end of the mercury thread which projects a short distance through and above the thin sheet rubber diaphragm that closes the top of the steam-point apparatus. To completely avoid this distillation, a simple device, called "the emergent steam heater" was added to the steam-point apparatus, by which the entire emergent part of the stem was kept at 100°. About two-thirds of the steam-point observations in the determinations of the fundamental intervals were made with the emergent stem heater. The value of the fundamental interval found in this way, however, does not differ appreciably from that found by the usual procedure, as the experiments referred to above show that, in the time required for two observers to make the necessary observations in steam, the somewhat lower reading at the steam point, due to neglecting the small stem correction for the exposed thread, just about compensates for the lowering of ice point (observed immediately after steam point) which is caused by the distillation of mercury. However, the convenience and satisfaction in working afforded by the emergent stem heater, and the fact that the ice-point readings obtained at different times can be better intercompared when distillation is avoided, has, in our opinion, fully justified its use.

It is possible that a somewhat greater concordance might be attained by the use of thermometers constructed of Jena 59^{III} borosilicate glass which has a zero depression about one-fourth that of verre dur. The resulting gain in accuracy would, however, not be as great as might be supposed. A careful study of the behavior of the thermometers in these intercomparisons gives evidence that the

factor which most limits the accuracy attainable in mercurial thermometry is the action of capillary forces, which manifest themselves especially in the determinations of the fixed points, where the resulting errors are not always eliminated by the limited number of observations usually made. This source of error, which is so troublesome when a stationary meniscus is being observed, is such as to render observations on a falling meniscus entirely untrustworthy. As a consequence of these forces the meniscus advances in a series of small and somewhat irregular steps, so that time-consuming micrometric measurements should be dispensed with, especially as the accuracy of estimation, for thermometers having fine graduations and an interval (0°.1) of 0.6 to 0.8 mm, is about 0°.002.

In conclusion the authors desire to acknowledge their indebtedness to Mr. E. F. Mueller, Mr. E. F. Wenderoth, and Mr. J. J. Crowe for their painstaking assistance in the work of computation and for all the barometer observations and reductions involved in the determinations of the fundamental intervals of the thermometers.

VI. APPENDIX.

CONSTANTS OF THERMOMETERS.

In the following pages are given the constants of the thermometers described in the present paper. The data concerning the dimensions of the thermometers, the description of the methods of calibration, and the tables of calibration corrections for the divisions at which these corrections were determined (every 2°) are taken from the certificates accompanying the thermometers, as furnished by the International Bureau. The remaining constants, viz, the external and internal pressure coefficient and the value of the fundamental interval of each thermometer, are those finally adopted as a result of the data obtained in the course of the present investigation as well as that given in the certificates.

CALIBRATION.

Division.—The equidistance of the divisions was examined with a dividing engine and found satisfactory.

Tonnelot Nos. 4331, 4332, 4334, 4335, 4336, 4623, and 4624.—Division into two parts of the interval [0°–100°] with three different threads (two threads with 4623 and 4624), observed 10 times in each of the positions [0.50] [50–100]. Division of the interval [0.50] into 5 parts by three (two for 4623 and 4624) independent calibrations, computed separately. Calibration every 2° of each 10° interval, prolonged 2° at each end (−2° to +12°, +8° to +22°, etc.; for 4623 and 4624, first interval *only* prolonged to −2°).

Baudin Nos. 15555 and 15583.—Division into two parts of the interval [0°–100°] with two different threads of 50°, observed 6 times in each of the positions [0.50] [50–100]. Division of the interval [0.50] into 5 parts. Calibration every 2° of each 10° interval, the first and last interval being prolonged to −4° and +52°, respectively.

Baudin Nos. 15282, 15962, 15963, and Tonnelot No. 11801.—Determination of corrections at 20°, 40°, 60°, and 80° by division into 5 parts, with threads of 20°, 40°, 60°, and 80°, observed 6 times in each position. Calibration every 2° of each 20° interval. Corrections at −4°, −2°, and +102°, +104° determined by supplementary calibrations.

Baudin Nos. 16016, 16017, and 16018.—Division into two parts of interval [0.100] with two different threads of 50°, observed 6 times in each of the positions [0.50] [50.100]. Division of the interval [50.100] into 5 parts. Calibration every 2° of each 10° interval, the first and last intervals being prolonged to 48° and 102°, respectively, the corrections for the divisions in the neighborhood of [0] were determined by supplementary calibrations.

Tables.—The corrections for intermediate points were obtained by graphic interpolation. The probable error of the corrections does not exceed ±0°.001.

PRESSURE COEFFICIENTS.

The values of the pressure coefficients (β_e and β_i) found in the present investigation, as well as those given in the certificates of the International Bureau, are given in Table I, page 678.

FUNDAMENTAL INTERVALS.

The values of the fundamental intervals of the thermometers found in the present investigation, as well as those given in the certificates, are given in Table VI, page 693.

SUPERCORRECTIONS TO MEAN VERRE DUR SCALE.

The supercorrections to reduce the scale defined by the thermometers (Tonnelot 11801, Baudin 15962, 16016, 16017, and 16018) to the mean verre dur scale of the Bureau International, as taken from the certificates, are given in Table XIX, page 717.

SUPERCORRECTIONS TO THERMOMETERS.

The supercorrections to reduce the scale defined by each thermometer to the mean scale defined by the 16 standard thermometers, as found from the intercomparisons already described, are given in Table XVIII, page 715.

Fig. 8.—Types of Thermometers.

TABLE XXI.

Constants of Thermometers.

Therm. No.	Maker	Date of construction	Range	Dimensions* A	B	C	Diam. of stem	Length of 1°
				mm	*mm*	*mm*	*mm*	*mm*
4331	Tonnelot..	July 1884	−4°.3 to + 51°.8 and 94°.2 to 104°.0 C.	62.0	448.7	518.5	4.7	6.560
4332	"	"	−4°.1 to + 51°.5 and 94°.7 to 103°.5 C.	60.0	461.3	527.0	5.0	6.908
4334	"	"	−4°.1 to + 51°.5 and 94°.3 to 103°.2 C.	60.0	461.3	527.5	5.0	6.866
4335	"	"	−3°.1 to + 52°.2 and 94°.9 to 103°.6 C.	55.0	461.0	529.0	4.8	6.926
4336	"	"	−3°.8 to + 51°.3 and 94°.2 to 103°.4 C.	60.0	463.8	533.0	5.0	6.910
4623	"	June 1888	−3°.3 to + 52°.0 and 95°.1 to 102°.6 C.	56.0	463.7	522.0	4.2	6.994
4624	"	"	−3°.6 to + 51°.7 and 95°.1 to 102°.5 C.	57.0	464.3	522.0	4.3	7.024
11801	"	Nov. 1895	−4°.7 to + 103°.1 C.	59.0	644.8	704.0	4.4	5.858
15282	Baudin ...	Apr. 1900	−5°.0 to + 104°.0 C.	68.0	659.3	725.0	5.1	5.913
15555	"	Oct. 1901	−4°.0 to + 52°.5 and 97°.5 to 102°.5 C.	63.0	539.7	606.0	4.7	8.252
15583	"	Dec. 1901	−4°.1 to + 52°.6 and 97°.4 to 102°.6 C.	63.0	529.3	602.0	4.9	7.954
15962	"	July 1903	−5°.1 to + 104°.1 C.	57.2	648.6	717.0	4.6	5.914
15963	"	"	−5°.1 to + 104°.1 C.	56.8	644.6	712.5	4.9	5.878
16016	"	Nov. 1903	−2°.6 to + 2°.6 and 47°.4 to 102°.6 C.	62.3	127.7	556.0	4.6	7.260
16017	"	"	−2°.6 to + 2°.6 and 47°.4 to 102°.6 C.	62.2	125.8	551.5	5.1	7.236
16018	"	"	−2°.6 to + 2°.6 and 47°.4 to 102°.6 C.	61.7	125.1	547.0	4.7	7.196

*The letters A, B, C refer to Fig. 8, on preceding page.

All of these thermometers are made of French hard glass (verre dur) and are graduated with equi distant divisions to $\frac{1}{10}$° C.

TABLE XXII.

Calibration Corrections.

Divs.	Tonnelot							Baudin	
	4331	4332	4334	4335	4336	4623	4624	15555	15583
−4								+0.0078	+0.0593
−2	+0.0292	−0.0039	−0.0047	−0.0545	−0.0052	+0.0119	+0.0253	+ .0025	+ .0305
0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
+2	− .0118	+ .0018	+ .0074	+ .0449	+ .0067	− .0129	− .0091	+ .0039	− .0184
4	− .0135	+ .0058	+ .0138	+ .0797	+ .0091	− .0227	− .0147	− .0022	− .0204
6	− .0224	+ .0089	+ .0151	+ .0982	+ .0095	− .0404	− .0197	− .0160	− .0085
8	− .0373	+ .0069	+ .0082	+ .0971	+ .0072	− .0546	− .0260	− .0274	+ .0030
10	− .0481	+ .0085	− .0059	+ .0881	+ .0005	− .0646	− .0343	− .0244	+ .0291
12	− .0569	+ .0081	− .0155	+ .0891	− .0108	− .0687	− .0430	− .0153	+ .0500
14	− .0629	+ .0117	− .0200	+ .1057	− .0240	− .0728	− .0516	− .0140	+ .0478
16	− .0622	+ .0210	− .0247	+ .1251	− .0360	− .0774	− .0583	− .0085	+ .0449
18	− .0521	+ .0289	− .0335	+ .1408	− .0448	− .0755	− .0595	− .0006	+ .0368
20	− .0316	+ .0223	− .0425	+ .1490	− .0519	− .0669	− .0577	+ .0007	+ .0236
22	− .0144	+ .0013	− .0529	+ .1401	− .0534	− .0587	− .0505	− .0040	+ .0091
24	+ .0041	− .0172	− .0653	+ .1271	− .0455	− .0533	− .0458	− .0058	+ .0010
26	+ .0173	− .0364	− .0726	+ .1238	− .0339	− .0498	− .0396	− .0101	− .0138
28	+ .0320	− .0564	− .0881	+ .1197	− .0195	− .0486	− .0303	− .0209	− .0255
30	+ .0391	− .0768	− .1076	+ .1138	− .0041	− .0474	− .0173	− .0332	− .0257
32	+ .0417	− .0986	− .1202	+ .0995	+ .0145	− .0395	− .0043	− .0298	− .0047
34	+ .0400	− .1268	− .1276	+ .0850	+ .0300	− .0299	+ .0053	− .0193	+ .0325
36	+ .0420	− .1482	− .1272	+ .0695	+ .0364	− .0193	+ .0150	− .0003	+ .0791
38	+ .0460	− .1499	− .1231	+ .0411	+ .0328	− .0108	+ .0195	+ .0127	+ .1033
40	+ .0420	− .1417	− .1117	+ .0164	+ .0281	− .0028	+ .0204	+ .0222	+ .0873
42	+ .0272	− .1317	− .0925	− .0049	+ .0283	+ .0054	+ .0132	+ .0264	+ .0620
44	+ .0125	− .1140	− .0768	− .0132	+ .0336	+ .0132	+ .0065	+ .0198	+ .0378
46	− .0010	− .0820	− .0610	− .0151	+ .0334	+ .0145	+ .0005	+ .0209	+ .0232
48	− .0056	− .0422	− .0325	− .0148	+ .0252	+ .0139	− .0005	+ .0187	+ .0159
50	+ .0037	+ .0056	+ .0012	− .0065	+ .0077	+ .0131	+ .0027	+ .0105	+ .0030
52				+ .0065				− .0010	− .0079
96	+ .0888	− .1062	− .0880	− .0248	+ .0279	− .0156	+ .0177		
98	+ .0477	− .0492	− .0478	− .0124	+ .0126	− .0042	+ .0138	+ .0048	− .0232
100	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
102	− .0384	+ .0516	+ .0552	+ .0178	− .0100	− .0114	− .0203	+ .0096	+ .0166

TABLE XXIII.

Calibration Corrections.

Divs.	Ton.	Baudin					
	11801	15282	15962	15963	16016	16017	16018
− 4		−0.0162	−0.0333	−0.0501			
− 2	+0.0169	− .0077	− .0208	− .0181	+0.0193	+0.0646	−0.0669
0	.0000	.0000	.0000	.0000	.0000	.0000	.0000
+ 2	− .0154	+ .0053	+ .0183	+ .0117	− .0160	− .0516	+ .0541
4	− .0278	+ .0118	+ .0328	+ .0104			
6	− .0267	+ .0022	+ .0422	+ .0100			
8	− .0219	− .0158	+ .0468	+ .0053			
10	− .0092	− .0290	+ .0428	− .0072			
12	− .0022	− .0294	+ .0396	− .0318			
14	+ .0067	− .0230	+ .0317	− .0525			
16	+ .0077	− .0175	+ .0298	− .0697			
18	+ .0096	− .0135	+ .0264	− .0761			
20	+ .0044	− .0101	+ .0282	− .0801			
22	+ .0033	− .0031	+ .0270	− .0833			
24	+ .0041	+ .0117	+ .0227	− .0859			
26	+ .0043	+ .0303	+ .0200	− .0857			
28	+ .0159	+ .0440	+ .0189	− .0820			
30	+ .0169	+ .0547	+ .0203	− .0738			
32	+ .0153	+ .0495	+ .0300	− .0640			
34	+ .0020	+ .0419	+ .0450	− .0566			
36	− .0171	+ .0366	+ .0625	− .0455			
38	− .0437	+ .0287	+ .0813	− .0343			
40	− .0479	+ .0179	+ .0939	− .0262			
42	− .0477	+ .0152	+ .1042	− .0163			
44	− 0324	+ .0118	+ .1092	− .0149			
46	− .0102	+ .0241	+ .1088	− .0202			
48	+ .0126	+ .0413	+ .1077	− .0152	+0.0386	+0.0511	−0.0395
50	+ .0293	+ .0581	+ .1055	− .0099	+ .0256	+ .0258	− .0143
52	+ .0392	+ .0563	+ .1004	− .0093	+ .0170	+ .0209	+ .0026

TABLE XXIII—Continued.

Calibration Corrections.

Divs.	Ton.	Baudin					
	11801	15282	15962	15963	16016	16017	16018
48	+0.0126	+0.0413	+0.1077	−0.0152	+0.0386	+0.0511	−0.0395
50	+ .0293	+ .0581	+ .1055	− .0099	+ .0256	+ .0258	− .0143
52	+ .0392	+ .0563	+ .1004	− .0093	+ .0170	+ .0209	+ .0026
54	+ .0502	+ .0509	+ .0960	− .0220	+ .0075	+ .0202	+ .0100
56	+ .0538	+ .0403	+ .0939	− .0397	+ .0025	+ .0170	+ .0082
58	+ .0596	+ .0352	+ .0887	− .0595	− .0024	+ .0175	− .0084
60	+ .0636	+ .0301	+ .0790	− .0790	− .0010	+ .0132	− .0273
62	+ .0651	+ .0209	+ .0685	− .0944	+ .0087	+ .0158	− .0448
64	+ .0720	+ .0175	+ .0573	− .1028	+ .0130	+ .0190	− .0590
66	+ .0800	+ .0083	+ .0492	− .1035	+ .0143	+ .0218	− .0696
68	+ .0809	− .0094	+ .0415	− .0986	+ .0145	+ .0190	− .0762
70	+ .0794	− .0365	+ .0335	− .1039	+ .0198	+ .0195	− .0827
72	+ .0813	− .0590	+ .0248	− .1070	+ .0209	+ .0092	− .0821
74	+ .0802	− .0765	+ .0162	− .1049	+ .0194	+ .0084	− .0803
76	+ .0922	− .0758	+ .0055	− .1015	+ .0198	+ .0135	− .0759
78	+ .0922	− .0739	− .0079	− .1010	+ .0180	+ .0157	− .0722
80	+ .0733	− .0755	− .0222	− .1032	+ .0159	+ .0163	− .0682
82	+ .0611	− .0726	− .0339	− .1101	+ .0157	+ .0118	− .0639
84	+ .0492	− .0643	− .0373	− .1035	+ .0085	+ .0076	− .0622
86	+ .0323	− .0519	− .0367	− .1011	+ .0009	+ .0017	− .0581
88	+ .0244	− .0315	− .0349	− .1021	.0000	+ .0011	− .0437
90	+ .0166	− .0100	− .0303	− .0907	+ .0037	+ .0039	− .0298
92	+ .0108	+ .0057	− .0249	− .0794	+ .0136	+ .0035	− .0146
94	+ .0044	+ .0178	− .0199	− .0593	+ .0137	+ .0099	− .0058
96	− .0025	+ .0259	− .0159	− .0365	+ .0117	+ .0066	− .0058
98	− .0013	+ .0175	− .0116	− .0107	+ .0062	− .0009	− .0021
100	.0000	.0000	.0000	.0000	.0000	.0000	.0000
102	+ .0188	− .0072	+ .0153	+ .0016	− .0120	− .0026	+ .0014
104		− .0083	+ .0311	− .0108			

O